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Biological power sources

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BIOLOGICAL POWER SOURCES

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INTRODUCTION

Statement of the Problem

Within the past five years several successful experiments have been conducted in which physiological data have been transmitted by radio from within the body to an external receiver. The kinds of information that can be handled by such systems are many; pulse rate, respiration rate, temperature, pressure and pH value are representative. These radio transmission methods have become known as physiological telemetry. The great advantage of telemetry is that no connecting wires or other similar constraints on the freedom of movement of the subject are necessary.

These telemetry methods generally can be classified into two types: 1) active systems which are powered by a small battery which is an integral part of the implanted package, and 2) passive systems which do not have an integral energy source but which are constructed in such a manner that they respond to an external energy source. The passive type units reported to date appear to have a very short transmission distance and to have only single channel operation. The active units, those not requiring external energy for operation, have generally made use of miniature batteries. The use of small energy storage batteries has placed upon systems so far reported a limit of approximately 20 feet on the transmission distances and often a relatively short time for obtaining useful information before the batteries fail. The battery lifetime limitation has been attacked with some success by utilizing pulse transmission systems with a small duty cycle and by using a battery that can be recharged by an external energy source. Generally, however, these limitations on the distance of transmission and the lifetime of the integral energy source are precluding many experiments of great interest and benefit to the advancement of knowledge in the biological and physiological sciences.

For radio telemetry systems specifically limited to medical applications, the possibility arises that the energy required might be extracted directly from biological or metabolic sources. The purpose of this research was to explore possible biological sources of energy sufficient for radio telemetry power requirements, to outline several procedures which might be used for converting this energy to the required electrical form, and to investigate in detail a particular conversion method.

Historical Background

Radio telemetry has long been a valuable tool of the electronics engineer for the study of phenomena which occur either at remote distances or at points where environmental conditions make direct observation difficult or impossible. The application of radio telemetry to physiological studies became important with the advent of miniaturized electrical components. These made possible the construction of radio transmitters small enough for attachment to or insertion within the body of research subjects. The reports of several early investigators demonstrated the feasibility of such systems and the number of reports and articles in current publication is indicative of the current interest and of the wide range of studies now in progress.

The earliest published account of the transmission of physiological data from within the body cavity appears to be that of Ardenne and Sprung (1) in 1956. They reported that a radio transmitter which could be swallowed was successfully used for the diagnosis of intestinal disturbances in man and indicated that such information as pressure, temperature and pH value could be measured with this system. The system was frequency modulated, used a 1.5 megacycle per second carrier and was powered by a nickel-cadmium cell yielding about 13 hours of useful operation. MacKay and Jacobsen (12) reported in 1957 a similar unit operating at 400 kilocycles per second which could transmit two measurements. They used a diaphragm to position a tuning slug in the oscillator coil to indicate pressure and the temperature coefficients of semiconductors to indicate temperature. The apparent battery lifetime was from two to four days.

In 1958, Marchal and Marchal (13) reported a passive unit they had used for some time. It appears to have been a quartz tuned circuit which, when pulsed by energy from an external source, responded by "ringing" the tuned circuit. The unit was used to take measurements in the digestive tract, and the authors claim it could be positioned in any part of the tract by means of a magnet.

The earliest reported use of a surgically implanted transmitter appears to be that of LeMunyan, et al (11) in January, 1959. They also utilized the kilocycle frequency range. The transmitter was capable of about 160 days operation to a maximum distance of about 18 feet. It was used to locate animals released for ecological studies. Other uses of surgically implanted active transmitters reported were the investigation of heart function by Kaeburn (10), November, 1959, and the investigation of the rumen pressure in a cow by Payne (17) in February, 1960. Further use of passive transmitters has been reported by Haynes and Witchey (7) in 1960. They used magnetic induction to ring a tuned circuit, the resonant frequency of the ringing indicating intestinal pressure.

A complete electrocardiography system employing a surgically implanted active transmitter was reported in 1960 by Essler (6). (This system was deemed of such significance that it was subsequently reported in the September 12, 1960, issue of Newsweek magazine, page 101.) The author indicated that the transmitter had a probable useful lifetime of 160 - 180 days. The animal subject was placed in a small pen in a laboratory room and the pickup antenna was a loop mounted on the walls of the pen, thus the transmission distance was probably not more than 10 feet. The transmitter was powered by a mercury cell.

The use of battery powered transmitters was furthered in 1961 by Douglas and Seal (4) who reported a system employing a rechargeable nickel-cadmium cell. The recharging system consisted of an external energy source operating at 80 kilocycles per second, a half-wave rectifier circuit inside the implanted package and a nickel-cadmium cell. The number of times the cell could be discharged and then recharged and retain suitable discharge characteristics was not specifically mentioned, but such a limitation is known to exist (16). The range of the transmitter was not reported, but apparently all experiments were performed inside a laboratory room.

From the work reported, a relatively low upper limit on the distance of transmission can be noted and often a short battery life is an important factor in limiting the duration of an experiment. These limitations are generally the result of the volume and weight maxima on implanted packages, for if more space and greater weight could be tolerated by the subject, then larger more powerful batteries could be used. It is probable that many investigations requiring operation beyond these limitations are being postponed awaiting either further advances in the energy storage capabilities of batteries or new and different types of energy sources.

APPROACHES INVESTIGATED

Introduction

The general method of approach to the study of biological power sources was that the energy required should be supplied wholly by the subject and that this energy should be available for the useful life of the subject or for the duration of the test period. Within this general method of approach, the following three subdivisions were considered: 1) existing biological potentials and chemical gradients; 2) blood pressure and blood flow; and 3) muscular activity and motion.

In order to evaluate the merits of any energy source device using these approaches, a typical telemetering transmission system was hypothesized and the requirements on the energy source determined from this system. The details of this calculation are presented in Appendix A. In summary, the proposed system was to operate at approximately 16 megacycles per second center frequency, have a useful range of 500 meters, have a signal to noise ratio greater than three decibels for a noise bandwidth of approximately 1.6 megacycles and a noise figure of 10, and be capable of this performance when implanted to a depth of 0.1 meters in animal tissue. From these considerations it was found that approximately one microwatt of direct-current power should be developed by the energy source.

Biological Potentials and Chemical Gradients

Biological potentials and chemical gradients were taken as a single method of approach to the problem because a device utilizing either would have many components and problems similar to a device using the other and also because both would be extracting energy from processes which are essentially chemical in nature. Those potentials and gradients specifically chosen for preliminary study were the nerve potentials, the stomach potentials, and the hydrogen ion concentration gradient existing between the stomach and the bladder.

The resting potentials and the action potentials of nerves have been studied by physiologists for many years. A summary of current knowledge concerning these potentials can be found in Stacy, et al (23). With regard to utilizing nerve potentials as a possible electrical power source, three difficulties are immediately evident. They are: 1) the voltage magnitude apparently rarely exceeds 100 millivolts which is a small value to be directly usable; 2) the quantity of energy extractable would be small since it must be replaced by the normal metabolic processes of the body; and 3) the direct extraction of electrical current (electrons) from such sources appears to be a formidable problem.

Another biological potential in the body is the potential which exists between the mucosa and serosa of the stomach and which is thought to be in some way connected to the production of hydrochloric acid in the stomach. Rehm (19), using the dog, has investigated this potential extensively and has reported that a potential of approximately 65 millivolts exists between the mucosa and serosa and that he found the electrical equivalent of the chemical energy production rate was about nine microwatts per square centimeter of stomach tissue area. The difficulties to be overcome in order to utilize this energy are: 1) the 65 millivolts voltage is a very small value to be directly usable; 2) the equivalent source internal impedance appears to be variable and often quite high; 3) suitable electrode systems would apparently be very complex and very large in size in terms of known components and techniques; and 4) permanent electrode contact with body tissues presents problems in tissue reaction.

The presence of chemical energy gradients such as the hydrogen ion concentration difference existing between the stomach and the bladder is a well known physiological fact. The difficulties in the utilization of this energy are principally those already mentioned for potentials.

With regard to the direct utilization of body potentials, it appears that electrodes used must either enter directly into a chemical reaction and thereby eventually be destroyed or leave toxic residues, or electrodes which are inert may be used with the attendant problems of supplying or removing gases which, among other things, would disturb previously established contact conditions.

Because of the difficulties in devising a suitable method for conversion of energy from the electron transport mechanisms of the body to that of electron flow in metallic conductors, the study of biological potentials and chemical gradients was not continued in detail.

Blood Pressure and Blood Flow

Considerable quantitative data are available concerning blood flow and blood pressure in the various animals. According to Dukes (5), the approximate total power output of the heart may be calculated with the following equation:

$$\text{Energy per minute} = \frac{7}{6}\,QR + \frac{mV^2}{g}\ \text{kg - m per min.}$$

where Q is the quantity of blood in liters

R is the resisting pressure in meters of blood

m is the mass of blood moved per minute in kilograms

V is the velocity of the blood flow in meters per second

g is the gravitational constant, 9.8 meters per second per second.

Calculations show that the total power output of the heart of a cow is about 12 watts, that of a man is about 8 watts and that of an average size dog is about 0.6 watts. Details of these calculations are presented in Appendix F. The diversion of as little as one per cent of this power would be more than sufficient for the purpose of powering the radio telemetry system proposed.

The Potter Instrument Company has constructed a device, The Potter Electroturbinometer (20), which can measure blood flow rate. This device and its associated instrumentation was designed and used to measure blood velocities by generating an electrical signal proportional to the blood velocity. Such a device might be converted to serve as an electrical power source. However, for permanent use, as the authors state, the problems of blood clotting caused by foreign bodies in the blood stream, hemolysis, and damage to the platelets by mechanical abrasion still remain. For short term use, the cumulative damage to the blood components may not be serious, and modern anticoagulents such as heparin could be used to inhibit clotting.

Because of the general sensitivity of the blood to foreign bodies and the sensitivity of the circulatory system to outside disturbances, the study of blood pressure and blood flow as a source of power was discontinued in detail.

Muscular Activity and Motion

Because the major problems associated with the direct conversion of biological energy to electrical energy appeared to be more physiological than engineering in nature, muscular activity and motion was selected for more intensive study and was the subject of the principal research effort. Two possible methods of utilizing muscular activity proposed were: 1) the use of a single muscle which could be detached at one end with no great inconvenience to the animal; and 2) the use of the complete bodily motion of the animal by means of an accelerometer type device.

The method of utilizing a single muscle detached at one end but in tact at its other contact point for nutrition and nerve connection was discontinued when it became evident that a feedback stimulating system would be complex and when the recovery rate of a muscle from contraction was found to be very slow for continual stimulation.

The utilization of the complete bodily motion of an animal was chosen as the method to be investigated in detail. An important advantage inherent in this method was that no direct interference with the body processes need be made. The device could be completely encased and the only physiological problem would be that of the tolerance of the body to a foreign object surgically implanted. Recent investigations of materials suitable for encapsulating such a device indicate that several materials are quite inert. A material known as "Silastic", developed by the Dow Corning Company, appears to be very suitable for this purpose (2).

Specifications

Because an accelerometer type device, a device inherently requiring a certain volume and mass, was to be considered, a study of volume and weight limitations for surgically inserted internal packages was made. A preliminary investigation led to the following specifications for an average size dog:

Maximum weight - approximately 200 grams.

Maximum dimensions - approximately 5 cm by 5 cm by 2 cm.

Selection of the accelerometer type device for detailed study and determination of these specifications on its physical size and weight together with its required power output of one microwatt completed the preliminary investigation of biological power sources.

POWER AVAILABLE FROM RANDOM MOTION

Introduction

In order to estimate the available power output of an accelerometer type device, a means of describing the motion of a subject animal in quantitative form was necessary. A preliminary investigation revealed that if an animal is subjected to random stimuli, such as would usually be found in its normal habitat, the animal's motion would, in all probability, also be random in nature. It was therefore proposed that a random type motion be used in this study. From studies of random signals by Wiener (25) and Middleton (15), it was found that a suitable mathematical description of a random motion could be made by specifying an autocorrelation function for the motion and, by direct analogy with their work, that an expression for the power available from the motion could be derived.

A General Power Integral

An autocorrelation function is defined as follows:

$$\phi(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} v(t)\, v(t + \tau)\, dt$$

where τ is the delay time, $2T$ is the correlation time of interest, $v(t)$ is the time description of the function, and $v(t + \tau)$ is the time description of the function when delayed by a time τ. If $v(t)$ is defined as the instantaneous velocity of a linear velocity damper, then, for a damping constant c, the average power dissipated in the damper is given by:

$$P = \frac{c}{2\pi} \int_{-\infty}^{\infty} |v(j\omega)|^2 \, d\omega$$

where ω is the radian frequency and $|v(j\omega)|^2$ is the square of the absolute value of the frequency spectrum of $v(t)$. The term $|v(j\omega)|^2$, often called the power spectral density, was found by noting that it is equal to the Fourier Transform of the autocorrelation function. The details of the development of this power integral are given in Appendix B.

One further modification of the power integral will considerably increase its usefulness. If the velocity function $v(t)$ is the result or output of some linear system operating on an input velocity, $v_{in}(t)$, which is Fourier Transformable, then the output power may be related to the input velocity. From linear system analysis it is known that

$$V(j\omega) = T(j\omega)\, V_{in}(j\omega)$$

where $V_{in}(j\omega)$ is the Fourier Transform of $v_{in}(t)$ and $T(j\omega)$ is the transfer function of the linear system. Then, since

$$|V(j\omega)|^2 = |T(j\omega)\, V_{in}(j\omega)|^2 = |T(j\omega)|^2 \, |V_{in}(j\omega)|^2$$

the power integral becomes

$$P = \frac{c}{2\pi} \int_{-\infty}^{\infty} |T(j\omega)|^2 \, |V_{in}(j\omega)|^2 \, d\omega \, .$$

The Autocorrelation Function and Squared Velocity Spectra

Once the assumption of random motion of the carrier had been made, several possible autocorrelation functions appeared to be feasible mathematical representations of this motion. Because calculations revealed little difference in the result but some difference in the ease of manipulation and computation, the often used exponential autocorrelation function was used here (See Appendix C). It is given by:

$$\phi(\tau) = \sigma^2 e^{-\omega_1 |\tau|}$$

where σ is the rms velocity and $\frac{1}{\omega_1}$ is the correlation time constant. A graph of $\phi(\tau)$ versus τ is shown in Fig. 1.

In Appendix B it is shown that

$$\mathcal{F}\left\{\phi(\tau)\right\} = |V(j\omega)|^2$$

where $\mathcal{F}\{\ \}$ indicates the Fourier Transform of the function in the braces. Therefore,

$$|V(j\omega)|^2 = \int_{-\infty}^{\infty} \sigma^2 e^{-\omega_1|\tau|} e^{-j\omega\tau} d\tau$$

$$= \sigma^2 \int_{-\infty}^{\infty} e^{-\omega_1|\tau|} (\cos \omega\tau - j \sin \omega\tau) d\tau$$ by Euler's equation. Because $e^{-\omega_1|\tau|} \cos \omega\tau$ is an even function and $e^{-\omega_1|\tau|} \sin \omega\tau$ is an odd function,

$$|V(j\omega)|^2 = 2\sigma^2 \int_0^{\infty} e^{-\omega_1 \tau} \cos \omega\tau \, d\tau = \frac{2\sigma^2\omega_1}{\omega_1^2 + \omega^2}$$

A graph of this function is shown in Fig. 2.

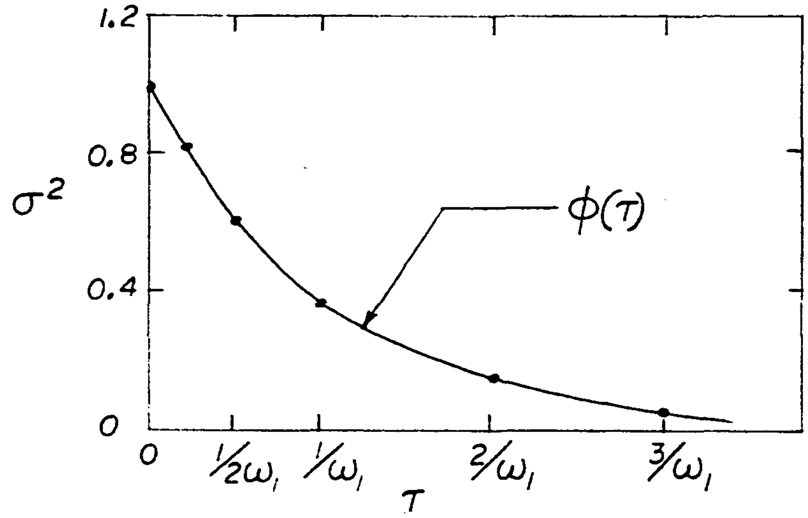

Fig. 1. $\phi(\tau)$ versus τ

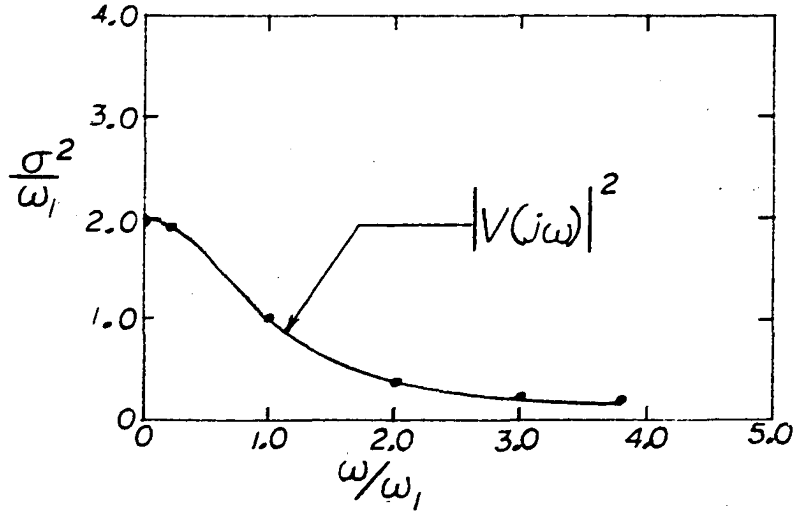

Fig. 2. $|V(j\omega)|^2$ versus ω

The Linear Transfer Function

The linear transfer function will depend upon the type of accelerometer system proposed. Two types which were investigated are described in the following material where they are designated as the Type I system and the Type II system.

The system designated as the Type I system, a simple mass, damper, and spring system, is shown diagramatically in Fig. 3. The differential equation describing this system is

$$m \frac{d^2y}{dt^2} + c \frac{dy}{dt} + ky = m \frac{d^2x}{dt^2}$$

Solving this equation by Laplace Transform methods yields

$$T(s) = \frac{s^2}{s^2 + \frac{c}{m} s + \frac{k}{m}}$$

$$T(j\omega) = \frac{-\omega^2}{(k/m - \omega^2) + j\omega \frac{c}{m}}$$

If critical damping is assumed as a special case, then these equations become

$$T(s) = \frac{s^2}{\left[s + \sqrt{k/m}\,\right]^2}$$

$$T(j\omega) = \frac{-\omega^2}{\sqrt{k/m} + j\omega}$$

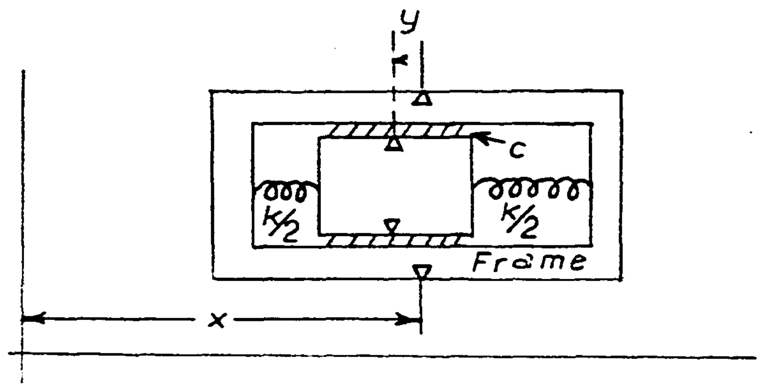

Fig. 3. A type I system

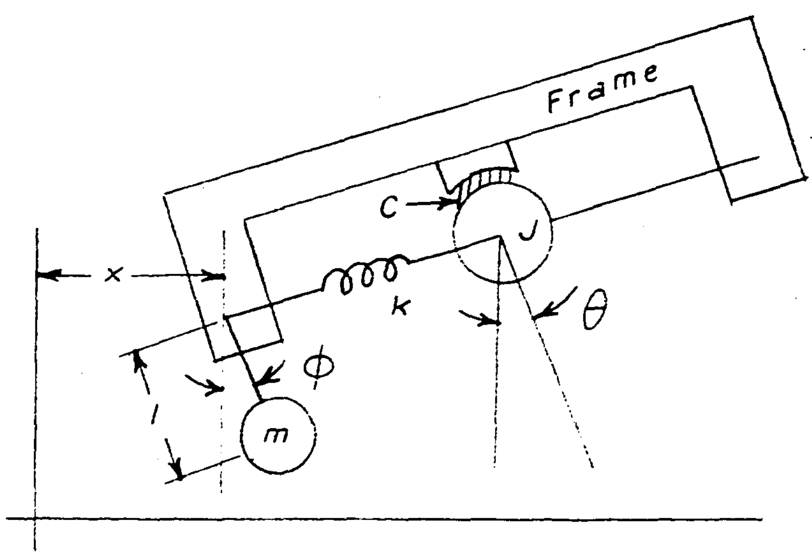

Fig. 4. A type II system

A more complex system, designated the Type II system, is shown in Fig. 4. This system is described by the simultaneous differential equations

$$J \frac{d^2\theta}{dt^2} + c \frac{d\theta}{dt} + k(\theta - \phi) = 0$$

$$m\ell^2 \frac{d^2\phi}{dt^2} + mg\ell\, \phi + k(\phi - \theta) = m\ell \frac{d^2x}{dt^2} \quad .$$

After solving by Laplace Transform methods, the transfer function is found to be

$$T(s) = \frac{k}{J\ell} \left[\frac{s^2}{s^4 + (\frac{c}{J})s^3 + (\frac{k}{J} + \frac{g}{\ell} + \frac{k}{m\ell^2})s^2 + (\frac{cg}{J\ell} + \frac{ck}{Jm\ell^2})s + (\frac{kg}{J\ell})} \right]$$

In order to simplify this equation for further manipulation, let

$$\alpha = \frac{c}{J} \qquad\qquad \beta = \frac{k}{J} + \frac{g}{\ell} + \frac{k}{m\ell^2}$$

$$\gamma = \frac{c}{J}(\frac{g}{\ell} + \frac{k}{m\ell^2}) \qquad\qquad \delta = \frac{kg}{J\ell}$$

Then

$$T(s) = \frac{k}{J\ell} \left[\frac{s^2}{s^4 + \alpha s^3 + \beta s^2 + \gamma s + \delta} \right]$$

$$T(j\omega) = \frac{k}{J\ell} \left[\frac{-\omega^2}{(\omega^4 - \beta\omega^2 + \delta) + j(\gamma\omega - \alpha\omega^3)} \right]$$

and

$$|T(j\omega)|^2 = \left(\frac{k}{J\ell}\right)^2 \left[\frac{\omega^4}{\omega^8 + (\alpha^2 - 2\beta)\omega^6 + (\beta^2 + 2\delta - 2\alpha\gamma)\omega^4 + (\gamma^2 - 2\delta\beta)\omega^2 + \delta^2}\right]$$

The Power Equations

The power integral, given on page 13 is

$$P = \frac{c}{2\pi} \int_{-\infty}^{\infty} |T(j\omega)|^2 \; |V_{in}(j\omega)|^2 \, d\omega.$$

By direct substitution, the available power from each type system is Type I

$$P_1 = \frac{c}{2\pi} \int_{-\infty}^{\infty} \left[\frac{\omega^2}{\left(\frac{k}{m} - \omega^2\right)^2 + \left(\frac{\omega c}{m}\right)^2}\right] \left[\frac{2\sigma^2 \omega_1}{\omega_1^2 + \omega^2}\right] d\omega$$

Type II

$$P_2 = \frac{c}{2\pi} \int_{-\infty}^{\infty} \left(\frac{k}{J\ell}\right)^2 \left[\frac{\omega^4}{\omega^8 + (\alpha^2 - 2\beta)\omega^6 + (\beta^2 + 2\delta - 2\alpha\gamma)\omega_1^4 + (\gamma - 2\delta\beta)\omega_1^2 + \delta^2}\right] \left[\frac{2\sigma^2 \omega_1}{\omega_1^2 + \omega^2}\right] d\omega$$

Both of these integrals are amenable to solution by the method of residues (8). Because of the quantity of algebra involved, the details of the solution are presented in Appendix D. In terms of the system parameters, the solutions are

$$P_1 = \sigma^2 \, m \, \omega_1 \, \frac{\omega_1\left(\frac{c}{m}\right) + \frac{k}{m}}{\omega_1^2 + \omega_1\left(\frac{c}{m}\right) + \frac{k}{m}}$$

$$P_2 = \sigma^2 m \omega_1 \, \frac{\left(\frac{g}{\ell} + \frac{k}{m\ell^2}\right)\omega_1^2 + \frac{c}{J}\left(\frac{g}{\ell} + \frac{k}{m\ell^2}\right)\omega_1 + \frac{kg}{J\ell}}{\omega_1^4 + \left(\frac{c}{J}\right)\omega_1^3 + \left(\frac{k}{J} + \frac{g}{\ell} + \frac{k}{m\ell^2}\right)\omega_1^2 + \frac{c}{J}\left(\frac{g}{\ell} + \frac{k}{m\ell^2}\right)\omega + \frac{kg}{J\ell}} \, .$$

In order to simplify the evaluation of these expressions, typical values for the parameters describing the random motion were taken as:

σ (rms linear velocity) = 1.0 foot per second or 0.305 meters per second

ω_1 (angular "cut-off" frequency) = 6.28 radians per second.

The limitations on the volume and weight of a practical device placed upper limits on the values of m, ℓ, and J. The following values were chosen as representative:

m (mass of the pendulum) = 0.1 kilogram, one-half of the total mass

ℓ (pendulum arm length) = 0.02 meters

J (rotational inertia = MR^2) = $(0.05)(0.02)^2 = 2 \times 10^{-5}$ kg - m^2.

The gravitational constant is 9.8 meters per second in the mks system of units employed here. As a result of these limitations and assumptions, the parameters c, the damping constant, and k, the spring constant, were the parameters which could be varied in order to maximize the power equations.

With regard to the result that only c and k may be varied greatly, the expressions for P_1 and P_2 were rearranged as follows:

$$P_1 = \sigma^2 m\omega_1 \left[\frac{c(\frac{\omega_1}{m}) + k(\frac{1}{m})}{\omega_1^2 + c(\frac{\omega_1}{m}) + k(\frac{1}{m})} \right]$$

$$P_2 = \sigma^2 m\omega_1 \left[\frac{(\frac{g\omega_1^2}{\ell}) + (\frac{\omega_1^2}{m\ell^2} + \frac{g}{J\ell})\, k + (\frac{g\omega_1}{J\ell})\, c + (\frac{\omega_1}{Jm\ell^2})\, kc}{(\omega_1^4 + \frac{g\omega_1^2}{\ell}) + (\frac{\omega_1^2}{m\ell^2} + \frac{g}{J\ell} + \frac{\omega_1^2}{J})k + (\frac{g\omega_1}{J\ell} + \frac{\omega_1^3}{J})c + (\frac{\omega_1}{Jm\ell^2})kc} \right]$$

Inspection of the expression for P_1 showed that both c and k should be as large as possible so that the bracketed term approached its limiting value of unity. The theoretical maximum for P_1 is then

$$P_{1\ max} = \sigma^2 m\omega_1 = (0.305)^2(0.1)(6.28) = 58.3 \text{ milliwatts}$$

for the parameter values given in the foregoing material. The maximum power available from a practical device will be determined by the values of expression, let c and k be related to each other by the following expression:

$$c^2 = \alpha^2 km.$$

Substitution of this expression into P_1 and dividing numerator and denominator by ω_1^2 gave

$$P_1 = \sigma^2 m\omega_1 \frac{\frac{\alpha\sqrt{km}}{m\omega_1} + \frac{k/m}{\omega_1^2}}{1 + \frac{\alpha\sqrt{km}}{m\omega_1} + \frac{k/m}{\omega_1^2}} = \sigma^2 m\omega_1 \frac{\alpha(\frac{\omega_0}{\omega_1}) + (\frac{\omega_0}{\omega_1})^2}{1 + \alpha(\frac{\omega_0}{\omega_1}) + (\frac{\omega_0}{\omega_1})^2}$$

where $\omega_0^2 = \frac{k}{m}$. A plot of P_1 versus $(\frac{\omega_0}{\omega_1})$ is shown in Fig. 5 for several values of α. If $\alpha^2 = 4.0$, then the system is said to be critically damped. This plot shows that, although theoretically c and k should be as large as possible for values of $(\frac{\omega_0}{\omega_1})$ greater than a value of approximately unity, the increase in available power may not be sufficient to make further practical refinements advisable because of the increasing difficulty in attaining these greater values of $(\frac{\omega_0}{\omega_1})$.

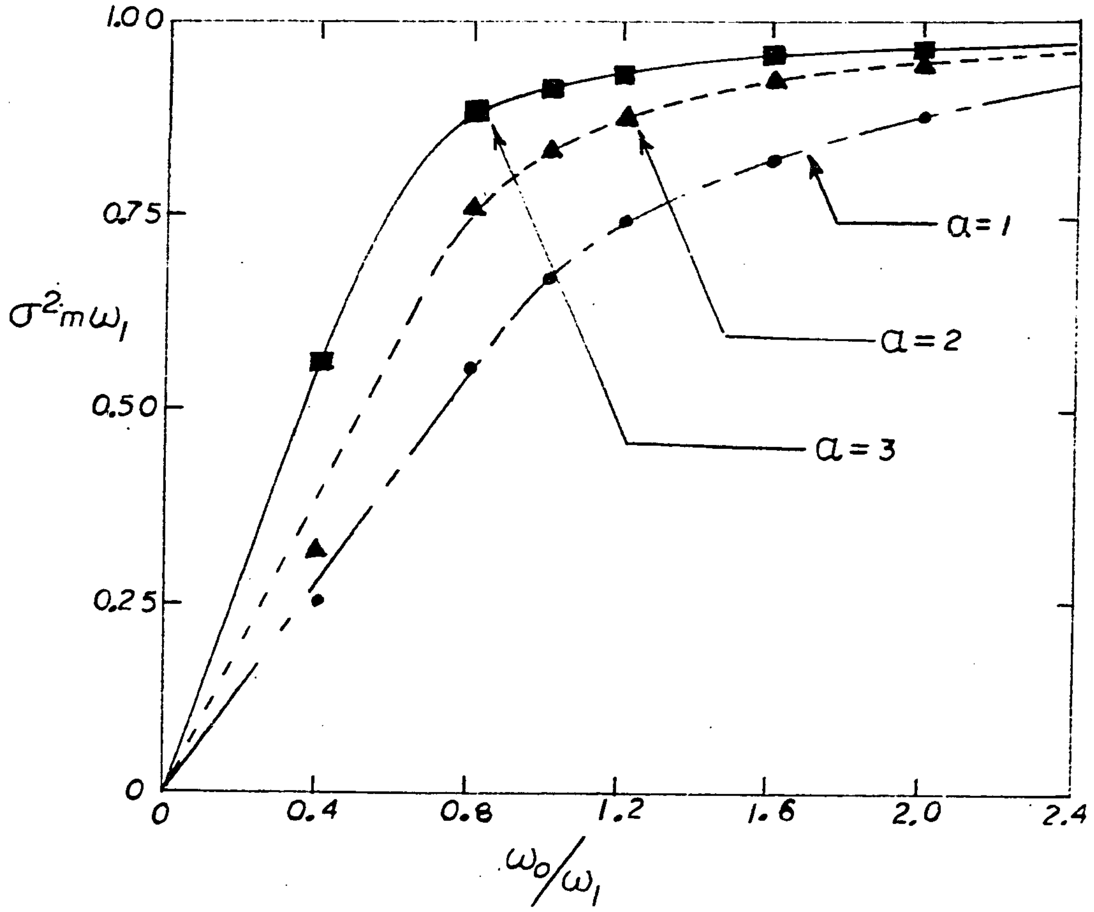

Fig. 5. P_1 versus $(\frac{\omega_0}{\omega_1})$

Inspection of the expression for P_2 showed that P_2 was almost completely insensitive to variations in c and k. Comparing the constants and the coefficients of c and k revealed the following:

$$\frac{\frac{g\omega_1^2}{\ell}}{\omega_1^4 + \frac{g\omega_1^2}{\ell}} = \frac{\frac{g\omega_1}{J\ell}}{\frac{g\omega_1}{\ell} + \frac{\omega_1^3}{J}} \doteq \frac{\frac{\omega_1^2}{m\ell^2} + \frac{g}{J\ell}}{\frac{\omega_1^2}{m\ell^2} + \frac{g}{J\ell} + \frac{\omega_1^2}{J}}$$

or simplified,

$$\frac{1}{1 + \frac{1}{\frac{g}{\omega_1^2\ell}}} = \frac{1}{1 + \frac{1}{\frac{g}{\omega_1^2\,\ell}}} \doteq \frac{1}{1 + \frac{1}{\frac{J}{m\ell^2} + \frac{g}{\omega_1^2\ell}}}$$

For the system parameters chosen, the term $\frac{J}{m\ell^2}$ is less than five per cent of $\frac{g}{\omega_1^2\ell}$. As a result, the complete bracketed expression can vary only between the value given by the foregoing ratio of coefficients and a maximum value of unity. This value of unity occurs when the quadratic term formed by the product of k and c is the controlling term in both the numerator and the denominator. Further inspection showed that this term was relatively insignificant for values of the product kc less than 10^{-5} and that it did become the controlling term for values of the product kc greater than 10^{-3}.

Typical values for k and c were derived from an analog computer simulation of the problem (See Appendix E). The analog computer used proved unsatisfactory for a complete solution because of excessive drift in the d-c amplifiers, but some data derived were suitable. The analog solution indicated that a $\frac{k}{J}$ value of approximately five was appropriate, thus an approximate value of $k = 5J = 5(2 \times 10^{-5}) = 10^{-4}$ for the system chosen. Similarly, a value of $\frac{c}{J}$ of approximately two was typical, thus a value of $c = 2J = 2(2 \times 10^{-5}) = 4 \times 10^{-5}$.

From this information, the quadratic term was found to be insignificant in the evaluation of P_2. Therefore, the value of P_2, the power available from the Type II system, was found to be

$$P_2 = (58.3 \times 10^{-3})(0.925) = 53.8 \text{ milliwatts}$$

for typical values as given in the foregoing material. Thus, regardless of the values of k and c, the power available from the Type II system would be not less than 53.8 milliwatts nor more than 58.3 milliwatts for the system parameters chosen.

Summary

The importance of these results to this investigation of biological power sources is found in the order of magnitude of the power delivered to a linear velocity damper. This magnitude of tens of milliwatts available should allow the delivery of one microwatt of d-c power to a transmitter with little difficulty, even though the conversion from mechanical to electrical power is very inefficient.

MECHANICAL TO ELECTRICAL CONVERSION

Introduction

The problem of converting the available mechanical power into the required electrical power was approached by two methods. They were: 1) the use of a permanent magnet rotor a-c generator, and 2) the use of a piezoelectric crystal. These two methods were to produce an alternating current which would then be rectified and the resulting direct current applied to a miniaturized transmitter. Consideration of practical oscillator circuits indicated that a d-c power of approximately 1.0 microwatt would be sufficient for a simple sine wave oscillator utilizing semiconductor devices. These would require a terminal voltage of approximately 0.4 volts at a load of approximately 10 microamperes. A maximum equivalent internal impedance of 3,000 ohms was specified although the importance of this impedance would depend primarily upon the type of oscillator circuitry employed.

The Permanent Magnet Generator

The study of the permanent magnet rotor a-c generator progressed through several stages from simple instrument magnets and coils of wire to complete generator construction. Three stages of investigation are represented by the models in Fig. 6. In Fig. 6a is shown a pendulum system with an instrument permanent magnet as the pendulum weight and a fixed coil of wire. As the magnet moved past the coil of wire, a narrow pulse of voltage of approximately 0.1 volt was generated. In Fig. 6b is shown

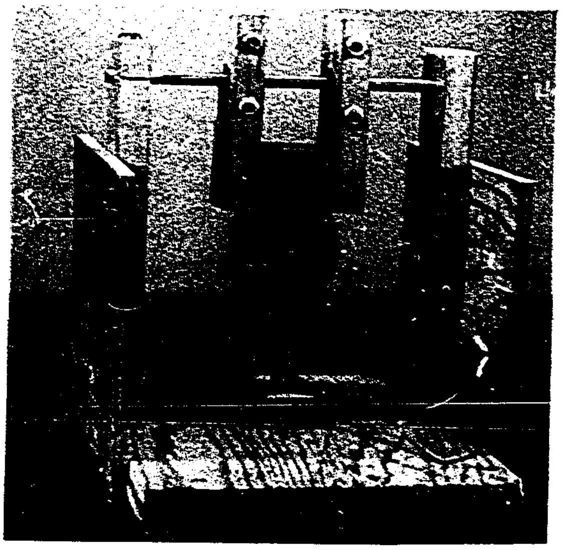

Fig. 6a. Pendulum and fixed coil

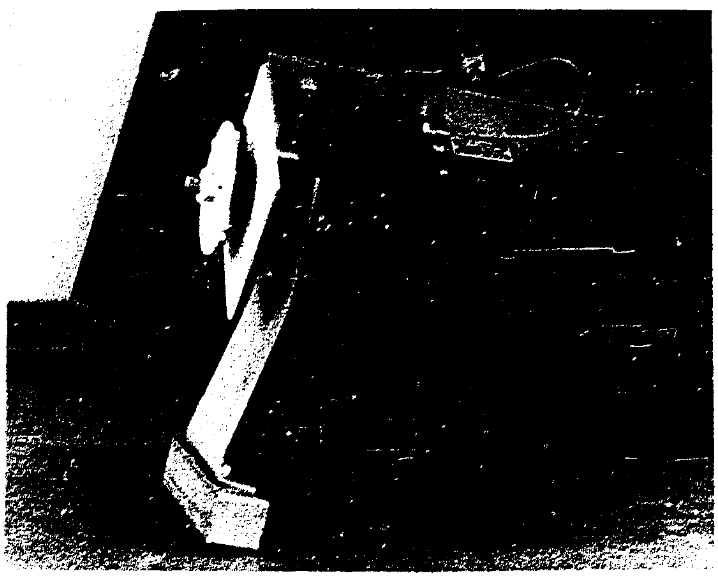

Fig. 6b. Pendulum and tachometer generator

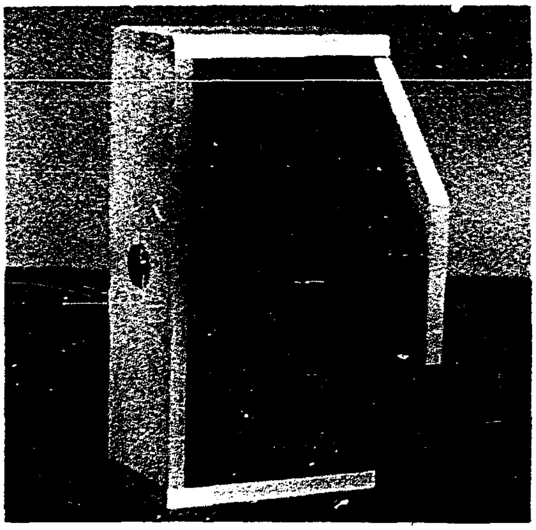

Fig. 6c. Type II system generator

a pendulum and gear train with a 11:1 ratio which drives a d-c tachometer generator. This system, when carried about or moved on the casters, generated about 50-100 microwatts but was unsatisfactory because of its size and the large amount of drag in the commutator brushes. In Fig. 6c is shown a complete generator utilizing a Type II system for increasing the relative velocity of the rotor and stator. A clock main spring was the torsional spring. It was very heavy and quite large.

A major difficulty with this permanent magnet method was the reduction of the total weight and volume of the generator while maintaining suitable flux paths for the magnetic fields; these are conflicting requirements. Another difficulty was the "locking-in" of the pole faces of the rotor on those of the stator. In order to maintain a relatively high flux density, the air gap between rotor and stator pole faces should be short. The shorter the air gap was made, the greater was the torque required to rotate the rotor from its "locked-in" position. Another difficulty was that the relative velocity of the stator and rotor was small, thus a large number of turns of copper wire would be required to generate the minimum voltage. The weight and volume limits conflicted with this requirement also.

A Type I system driving a generator would experience all of these difficulties, excessive weight and volume, slow relative velocities, and a large number of turns required. The Type II system was an attempt to increase the relative velocity of the rotor and stator of a generator by means of energy stored in the rotational spring. The "locking-in" of the rotor was an advantage, although a small one, in that the rotor would not rotate until the pendulum shaft had rotated through some angular displacement.

When the rotor did move, it moved rapidly to the adjacent or to the second stator pole position and "locked-in" in that position. This produced a higher voltage but for a shorter interval of time. The internal impedances of both systems, although they were not measured, would presumably be less than the maximum of 3,000 ohms specified.

An additional factor, that of inherent frictional losses, would probably become increasingly important in further miniaturization than that attempted. Because of these difficulties, a final miniaturization of a permanent magnet rotor generator was not attempted.

The Piezoelectric Crystal Converter

Piezoelectric crystals of various materials and a variety of "cuts" have been used as electro-mechanical transducers for some time (14). These crystals can be obtained easily and are not usually expensive. The major advantage offered to this investigation by piezoelectric crystals was their light weight and small volume.

The output voltage of a piezoelectric crystal can be made very high. A value of 20,000 volts has been reported by the Clevite Corporation for a special ceramic type (3). A typical phonocartridge crystal will deliver 4-5 volts into an open circuit or very high impedance. The equivalent source impedance of a piezoelectric crystal is very high, normally 100,000 ohms or higher, presenting an impedance matching problem in circuits requiring a low impedance source. A typical phonocartridge of the type investigated in this research is shown in Fig. 7 and the internal mountings and the crystal are shown in Fig. 8. It can be seen from Fig. 8 that the

Fig. 7. Phonocartridge

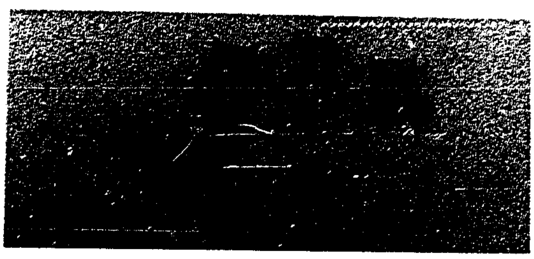

Fig. 8. Cartridge components

crystal itself is mounted in soft rubber. Thus, the efficiency of conversion of the mechanical torque to electrical power was low.

Measurements of the d-c power output of a phonocartridge were made by passing the cartridge needle lightly over an unpolished but smooth surface and rectifying the output with a full-wave semiconductor diode bridge. It was determined that the crystal could deliver approximately 15 microwatts to a matched load. The output was an alternating voltage containing frequencies to well above 1,000 cycles per second. Impedance matching was then possible with a miniature input transformer, the one chosen being a 100K to 1K air core transistor input transformer. A completely satisfactory d-c voltage of approximately 0.5 volts at a d-c current of approximately 30 microamperes was delivered when the crystal was moved lightly over the test surface. Theoretically, this voltage and current output level would be sufficient to power a semiconductor oscillator.

Summary

As a result of the preliminary investigation and evaluation of the proposed methods of conversion from mechanical to electrical power the piezoelectric crystal was selected for further study. The output voltage and current were found to be sufficient, theoretically, to operate an oscillator and the approximate source impedance of 1,000 ohms was less than the previously determined maximum of 3,000 ohms. Its small size and weight were also important advantages.

A TEST MODEL

Introduction

The proposal to utilize a piezoelectric crystal converter was tested for feasibility by constructing a converter and attaching an oscillator circuit to the output of the converter. As this was to be a feasibility study only, no attempt to miniaturize the phonocartridge assembly was made although it can be seen from Fig. 8 that the overall length could probably be reduced by one-half by suitable changes in the torque-transmitting yoke and the two electrical output connections.

Description of the Test Model

The test model consisted of essentially two parts, the converter assembly and the oscillator. A photograph of the complete converter and oscillator test model is shown in Fig. 9.

The test converter was constructed, in order of electrical connection, of the phonocartridge, a 100K to 1K impedance matching transformer, a semiconductor diode bridge rectifier and a 47 microfarad capacitor connected to the output of the rectifier. As an unmodified phonocartridge was used, a phonograph needle was employed to transmit the mechanical displacement of the frame to the crystal.

The test oscillator circuit employed a Hoffman HU-10 Uni-tunnel diode in series with an inductor. This Uni-tunnel diode exhibits a negative resistance region similar to that of a tunnel diode except that it occurs at a nominal current peak of 10 microamperes. This oscillator was

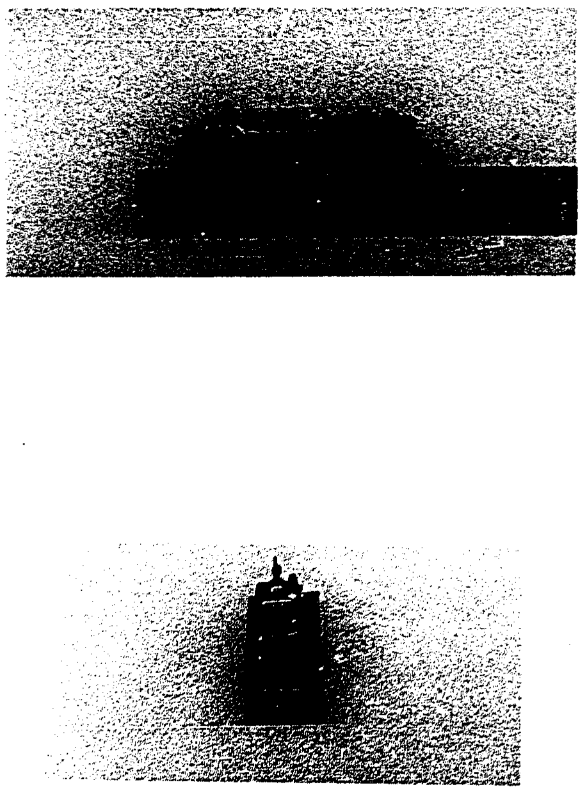

Fig. 9. The test model

then a simple negative resistance sine-wave oscillator whose frequency of oscillation was approximately that of the series resonant frequency of the inductor and the junction capacitance of the diode. The radio frequency output power of this oscillator was found by using the following approximate formula for silicon devices:

$$P_0 = \frac{I_p}{28} \text{ watts where } I_p \text{ is the peak value of the current at beginning of the negative resistance region.}$$

A complete discussion of tunnel diode circuitry and a development of the foregoing formula are given in the Tunnel Diode Manual (24).

Characteristics of the Test Model

The characteristics of the HU-10 diode were checked by using the Tektronix Transistor Curve Tracer. The peak current was found to be six microamperes so that the approximate power output at a resonant frequency of approximately two megacycles per second was

$$P_0 = \frac{I_p}{28} = \frac{6 \times 10^{-6}}{28} \doteq 0.22 \text{ microwatts.}$$

The d-c input power was approximately, for V and I in the negative resistance region of the diode,

$$P_{in} = V_{dc}\, I_{dc} = (0.12)(5 \times 10^{-6}) = 0.72 \text{ microwatts.}$$

Thus, the conversion efficiency of the oscillator was approximately 30 per cent. A photograph of the oscilloscope presentation of the oscillator waveform is shown in Fig. 10. It can be seen that the waveform contains

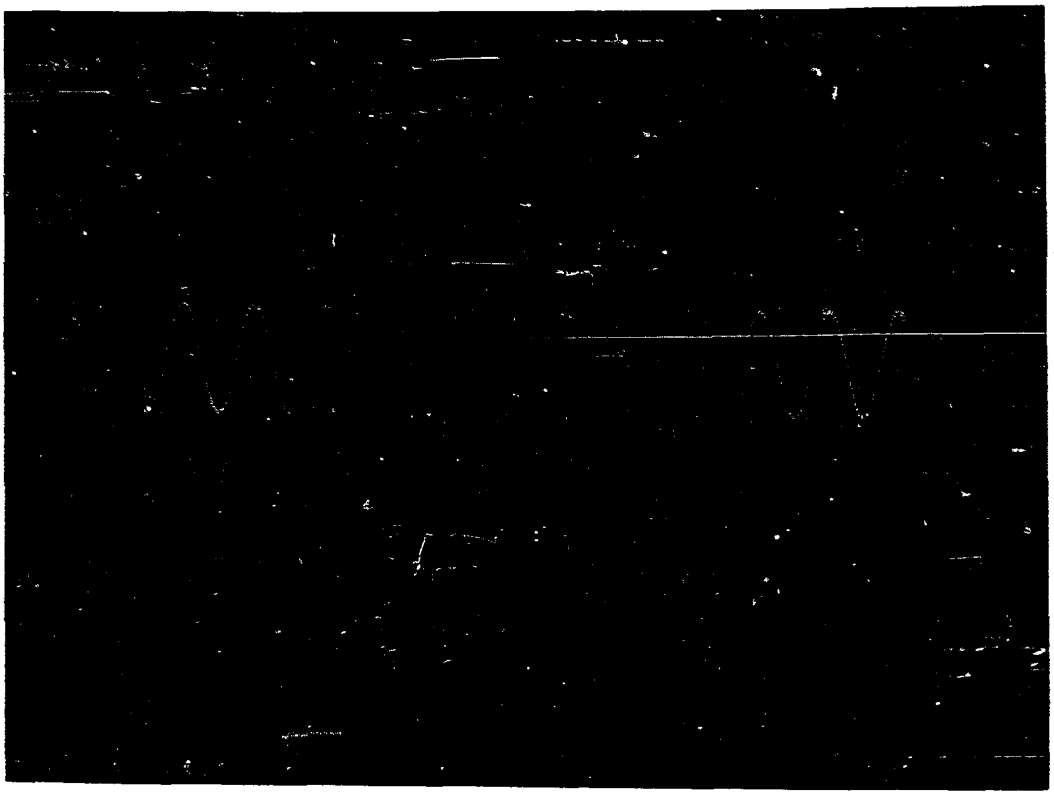

Fig. 10. Oscillator waveform

no large harmonic frequency components and is quite frequency stable. A test of the frequency and amplitude stability showed that for a voltage of from 0.11 volts to 0.14 volts applied to the diode, no significant changes could be found. Oscillations were sustained for voltages from 0.10 volts to 0.15 volts, but at the extremes both frequency and amplitude varied widely. The electrical circuit representation of the complete test model is shown in Fig. 11.

Summary

The feasibility of operating an oscillator from the power supplied by a piezoelectric crystal was demonstrated by the test model. The converter was found to be light in weight, small in size and sufficiently powerful to yield sustained oscillations using the test oscillator. This result completed the investigation of biological power sources.

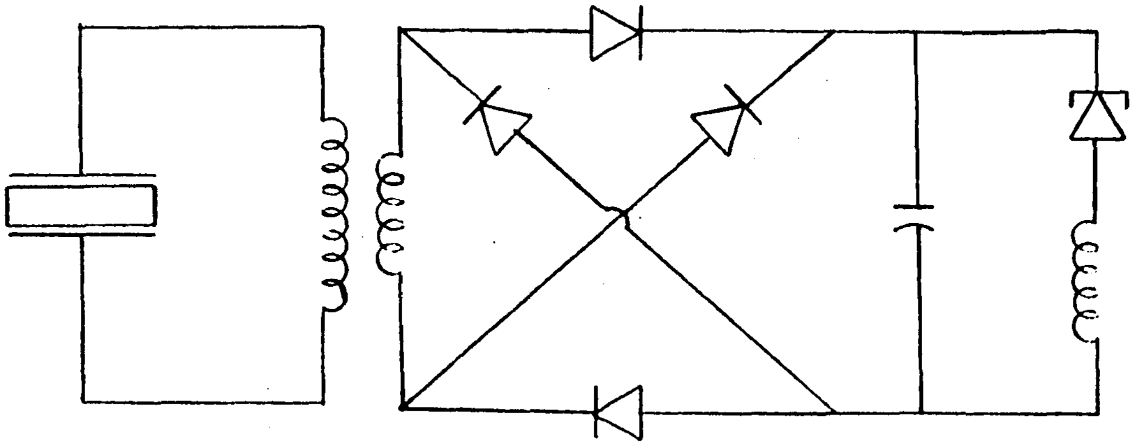

Fig. 11. Test model schematic

CONCLUSIONS AND RECOMMENDATIONS

Review of the Results

The results of the investigation of biological power sources indicated that at least one method, the use of a piezoelectric crystal for converting the energy of motion into electrical energy, is a feasible method for powering physiological telemetering systems. Because there are no batteries to replace, this method becomes even more important for those telemetry systems which employ surgically implanted transmitters.

The preliminary investigation of the biological potentials and chemical gradients indicated that at present, the electrode problems are very troublesome, but that these sources may become useful if and when this electrode problem is solved.

The use of the energy of the circulatory system, blood pressure and blood flow, also has promise but, at present, the damage to the blood components is the primary reason for not utilizing this energy. As the investigations of new materials continues, it is possible some may be found that will permit long term use of devices inserted directly into the blood vessels with only small, tolerable amounts of damage to the circulatory system.

Suggestions for Improving the Converter

The converter used to demonstrate the feasibility of a transmitter powered by biologically supplied energy could be improved in several respects. Among the most important of these are:

1) the size could be reduced considerably by designing a much more compact torque transmitting lever and yoke and by a much more compact system of output electrical connections;
2) much more active crystals than the Rochelle Salt type are available, of special interest are the new ceramic types;
3) perhaps a bimorph (simple lever or "leaf spring" type) crystal would be more suitable for many purposes;
4) a method of voltage regulation, perhaps using the junction voltage of a semiconductor diode as a reference, would provide greater stability (the test model should be regulated at 0.12 volts, the source often supplied more than this and the oscillations ceased until the voltage again dropped to the negative resistance region of the diode).

Some Possible Uses for the Converter

The particular problems for which this converter could be used would, in part, determine the mechanical system employed to activate the crystal. Two possible uses, utilizing two different mechanical systems, might be measurement of the respiration rate and measurement of the linear acceleration of a subject.

Respiration rate might be measured by a system which used the expansion and contraction of the thoracic cavity to operate a bellows type device which in turn could cause activation of the crystal. The crystal could be held in place by the rib cage. As the lungs expanded and contracted, a voltage would be generated and a pulse of radio frequency energy would be transmitted, one pulse upon expansion and another upon contraction.

Linear acceleration might be measured by a Type I system and the piezoelectric converter. When acceleration or deceleration occured, the oscillator would be activated and a signal transmitted.

The basic requirement for these converter systems is that relative motion between two points or two surfaces be present. The crystal itself can be made to vibrate at a much faster rate than the relative motion in several ways. For example, the crystal could be moved over a surface which is very slightly roughened or which has a predetermined groove similar to a phonograph record groove cut in the surface. These are not difficult requirements because the required strain on a piezoelectric crystal is in the order of thousandths of an inch.

In conclusion, the feasibility of operating a telemetry oscillator without batteries was shown. The mechanical motion to electrical energy converter developed was shown to be useful whenever any type of relative mechanical motion was available as well as for the random motion accelerometer system originally proposed. The power source developed in this investigation would seem to have great promise for use in future physiological telemetry systems.

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APPENDIX A

Power Requirements for Physiological Telemetering

A transmitter implanted within animal tissue and transmitting information to a receiver located in air at some distance from the receiver must develop its radiated power into a lossy medium, that of animal tissue. The electrical properties of tissue, for radio frequencies less than about 500 megacycles per second are generally taken to be those of a homogeneous saline solution similar to salt water. Above about 500 megacycles, the electrical properties of skin, fat, and the deep tissues become sufficiently dissimilar that the assumption of homogeniety is no longer valid. (21) The telemetry systems proposed all were to operate at frequencies considerably below 500 megacycles. Therefore, tissue was assigned the following physical constants: $\mu = 4\pi \times 10^{-7}$ henrys per meter; $\epsilon = \frac{10}{36\pi \times 10^{9}}$ farads per meter; and $\sigma = 4$ mhos per meter. These constants are respectively the permeability, the permittivity or dielectric constant, and the conductivity of the medium. The remainder of the transmission path was to be in air and the accepted values of these physical constants for air is: $\mu = 4\pi \times 10^{-7}$ henrys per meter; $\epsilon = \frac{1}{36\pi \times 10^{9}}$ farads per meter; and $\sigma = 0$ mhos per meter.

Jordan (9) gives the following equations for describing the propagation of plane waves of electromagnetic energy in a generalized medium:

$$\gamma \text{ (propagation constant)} = (j\omega\mu\sigma)(1 + j\frac{\omega\epsilon}{\sigma}) \quad \text{per meter}$$

$$\eta \text{ (intrinsic impedance)} = \frac{j\omega\mu}{\sigma + j\omega\epsilon} \quad \text{ohms}$$

where ω is the radian frequency of the radio wave and j is the complex variable operator. For air, with $\sigma = 0$, these reduce to

$$\gamma_{air} = j\omega \quad \mu\epsilon \qquad\qquad \eta_{air} = \frac{\mu}{\epsilon}$$

and for tissue, since $\frac{\sigma}{\omega\epsilon} \ll 1$ for the frequencies considered,

$$\gamma_{tissue} = j\omega\mu\sigma = \frac{\omega\mu\sigma}{2} (1 + j1) \text{ per meter}$$

$$\eta_{tissue} = \frac{j\omega\mu}{\sigma} = \frac{\omega\mu}{2\sigma} (1 + j1) \text{ ohms.}$$

The reduction in strength of the electromagnetic field can be described, for a plane wave, by the following equations, also in Jordan (9)

$$E_2 = E_1 e^{-\alpha \ell} \; , \; E_4 = E_3 \frac{2\eta_4}{\eta_3 + \eta_4}$$

where E is the electric field intensity in volts per meter, α is the attenuation (real part of γ) in nepers per meter, and the subscripts 1,2 refer to two points in the same medium and the subscripts 3,4 refer to two boundary points in the different media, media 3 and media 4. Further reduction in the field strength occurs because the radio waves will not be plane waves but will be concentrated in one general direction by the antenna system employed. The system with minimal concentration is the isotropic antenna system, one which radiates equally in all directions in 3-dimensional space. For this least desirable system, the power densities in watts per square meter at various distances are related by (9) $P_2/P_1 = d_1^2/d_2^2$ and the power density at a distance d is related to the radiated power by $P_d = W_{rad}/4\pi d^2$. As a result, radiated power is attenuated by both "spreading" of the field and by losses in the medium through which it is propagated.

To approximate the attenuation resulting from the lossy medium, let the thickness of tissue be 0.1 meters and the frequency be 15.9 megacycles per second, giving an ω of 10^7 radians per second. Then

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \left[\frac{(10^7)(4\pi \times 10^{-7})(4)}{2}\right]^{1/2} = \sqrt{8\pi} \ \text{nepers/meter}$$

$$\alpha\ell = 0.1\ \sqrt{8\pi} \doteq 0.5 \text{ nepers and } E_2 = E_1 e^{-0.5} \doteq (\tfrac{2}{3})E_1 .$$

That this approximation from plane wave theory is reasonably valid for short transmission distances in tissue is verified by an experiment performed by Douglas and Seal (4). At 100 megacycles, they found that signal strength was reduced to 75 per cent of that in free air when the transmitter was implanted inside the body cavity of a dog. It thus appears that the calculations presented may be somewhat pessimistic in the predicted results, but, though the thickness of tissue used in the experiment was not specified, it appears to have been less than the 0.1 meters assumed in the calculations.

The attenuation in the tissue due to the spherical wave front is, for power density at the 0.1 meter distance

$$P_{0.1_m} = \frac{W_{rad}}{4\pi(0.1)^2} = 8\ W_{rad} \ \text{watts/meter}^2 .$$

In terms of the electric field intensity at any point, the power density at that point may be written

$$P_d = \frac{E^2 \cos \theta}{2\,|\eta|} \text{ watts/meter}^2$$

where E is the instantaneous peak value of the electric field intensity and cos θ is the power factor for the medium. At the boundary between tissue and air, the electric field changes in value because of reflections due to the different intrinsic impedances of air and tissue. These impedances are:

$$\eta_{air} = \sqrt{\frac{\mu}{\epsilon}} = 120\pi \text{ ohms}, \quad \eta_{tissue} = \sqrt{\frac{\omega\mu}{\sigma}} \;\underline{/\,45^{\circ}} \text{ ohms.}$$

Therefore,

$$\frac{E_{air}}{E_{tissue}} = \frac{2\,\eta_{air}}{\eta_{tissue} + \eta_{air}} \doteq 2$$

From the above calculations, including both attenuating factors,

$$E^2_{tissue\ 0.1} = \frac{2|\eta|\ P_d}{\cos \theta} \left(\frac{4}{9}\right) = \frac{64\sqrt{2\pi}\ W_{rad}}{9}$$

Then $E^2_{air} \doteq 4\ E^2_{tissue} = \frac{256\sqrt{2\pi}}{9} W_{rad}$

and $P_{0.1_m} = E^2/\eta_{air} = \frac{256\sqrt{2\pi}}{1080\ \pi}\ W_{rad}$

or finally $P_{0.1_m} \doteq 0.1\ W_{rad}$ watts/meter^2 .

To complete the calculation, for any point d in air, d in meters, the power density is given by

$$P_d = (0.1\ W_{rad}) \left(\frac{0.1}{d}\right)^2 = \frac{W_{rad}}{1000\ d^2} \text{ watts/meter}^2 .$$

For satisfactory reception of the information, the signal to noise ratio should be greater than three decibels where the decibel level is given by $S/N = 10 \log_{10} \frac{P_r}{W_n}$ decibels where W_n is noise power and P_r is the received power, both in watts. W_n, the effective noise power, is found from the following (18)

$$W_n = k\ T\ B\ F \text{ watts}$$

where k (Boltzman's constant) = 1.38×10^{-23} joules per degree K

T = Temperature in degrees K (Kelvin), typically 300° K

B = effective noise bandwidth, typically 0.1 center frequency

F = noise figure, typically about 10.

Thus $W_n = (1.38)(300)(1.59)(10)(10^{-23})(10^{6}) \doteq 10^{-14}$ watts.

From this calculation, the minimum received power is

$$3 = 10 \log_{10} \left(\frac{P_r}{10^{-14}}\right)$$

$$P_r = 10^{-14} \left(10^{+0.3}\right) \doteq 2 \times 10^{-14} \text{ watts.}$$

To refer this minimum power to a typical receiving system, consider a half-wave dipole in free space for which the received power is given by (9) $P_r = A\ P_d$ where A is the effective aperture of the antenna and P_d is the power density at the antenna. The effective aperture for this antenna is $1.31\ \lambda^2$ where λ is the wavelength. The wavelength for a 15.9 megacycle radio wave is approximately 20 meters. Thus

$$P_r \varnothing (1.31)(20)^2\ P_d = 524\ P_d \text{ watts}$$

or, from the above calculati ons,

$$P_d = \frac{P_r}{524} = \frac{2 \times 10^{-14}}{524} \doteq 4 \times 10^{-17} \text{ watts/meter}^2 .$$

If d is taken as 500 meters, then the required radiated power is

$$\omega_{rad} = 1000\ P_d\ d^2 = (10^3)(4 \times 10^{-17})(25 \times 10^4) = 10^{-8} \text{ watts.}$$

In order to determine the quantity of d-c source power necessary to radiate the above theoretical minimum power, consideration must be given to the efficiency of coupling from the oscillator to the medium supporting the radiation and to the efficiency of the oscillator in converting the d-c source power to radio frequency energy. Since it is conceivable that total over-all efficiency might be less than ten per cent, a minimum d-c source power of approximately one microwatt should be available.

APPENDIX B

A General Power Equation

Let a velocity function be defined as follows:

$$v_T(t) = v(t) \qquad -T \leq t \leq T$$

$$v_T(t) = 0 \qquad \text{elsewhere}$$

where $v(t)$ is defined for all t. Then

$$\phi_T(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} v_T(t)\, v_T(t+\tau)\, dt$$

and

$$\phi_T(\tau) = \frac{1}{2T} \int_{-\infty}^{\infty} v_T(t)\, v_T(t+\tau)\, dt$$

from the definition of $v_T(t)$. The Fourier Transform of $\phi_T(\tau)$ is then:

$$\mathcal{F}\left\{\phi_T(\tau)\right\} = \frac{1}{2T} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} v_T(t)\, v_T(t+\tau)\, dt \right) e^{-j\omega\tau}\, d\tau$$

$$= \frac{1}{2T} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} v_T(t)\, v_T(t+\tau) e^{j\omega t}\, e^{-j\omega(t+\tau)}\, dt \right) d\tau$$

If $s = t + \tau$, then $ds = d\tau$, and

$$\mathcal{F}\left\{\phi_T(\tau)\right\} = \frac{1}{2T} \left(\int_{-\infty}^{\infty} v_T(s) e^{-j\omega s}\, ds \right) \left(\int_{-\infty}^{\infty} v_T(t) e^{j\omega t}\, dt \right)$$

Thus:

$$\mathcal{F}\left\{\phi_T(\tau)\right\} = \frac{1}{2T} V_T(j\omega)\, V_T^*(j\omega) = \frac{1}{2T} \left|V_T(j\omega)\right|^2$$

where $V_T^*(j\omega)$ is the complex conjugate of $V_T(j\omega)$.

Also, $V_T(j\omega)$ and $v_T(t)$ form a transform pair because

$$\mathcal{F}\left\{v_T(t)\right\} = \int_{-\infty}^{\infty} v_T(t)e^{-j\omega t}\,dt = \int_{-T}^{T} v(t)e^{-j\omega t}\,dt = V_T(j\omega)$$

If $v_T(t)$ is the velocity of the damping element of a mechanical system, the instantaneous power dissipated in the damper is $p_T(t) = c|v_T(t)|^2$ where c is the damping constant of the system. Also, from the definitions of $v_T(t)$ and $V_T(j\omega)$,

$$v_T(t) = \mathcal{F}^{-1}\left\{V_T(j\omega)\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} V_T(j\omega)\, e^{jt\omega}\,d\omega.$$

Thus, the average power dissipated in the damping over the period 2T seconds is:

$$P_T = \frac{1}{2T}\int_{-T}^{T} c|v_T(t)|^2\,dt = \frac{c}{2T}\int_{-T}^{T} v_T(t)\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} V_T(j\omega)e^{jt\omega}\,d\omega\right)dt.$$

As both integrals are absolutely convergent, the order of the integration may be interchanged, and

$$P_T = \frac{c}{(2T)(2\pi)}\int_{-\infty}^{\infty} V_T(j\omega)\left(\int_{-T}^{T} v_T(t)\, e^{j\omega t}\,dt\right)d\omega$$

$$= \frac{c}{2\pi}\int_{-\infty}^{\infty} \frac{V_T(j\omega)\; V_T^*(j\omega)}{2T}\,d\omega.$$

Therefore,

$$P_T = \frac{c}{2\pi}\int_{-\infty}^{\infty} \frac{|V_T(j\omega)|^2}{2T}\,d\omega\,.$$

To extend the use of this equation to include v(t), defined for all t, requires an additional restriction. The term $\frac{|V_T(j\omega)|^2}{2T}$ is, dimensionally, average power density per unit damping over the frequency interval $d\omega$. If v(t) belongs to a time stationary process, then the average power density of v(t) is given by (8)

$$|V(j\omega)|^2 = \lim_{T \to \infty} \frac{1}{2T} |V_T(j\omega)|^2$$

Then, average power is given by

$$P = \frac{c}{2\pi} \int_{-\infty}^{\infty} |V(j\omega)|^2 \, d\omega.$$

Because $V(j\omega)$ will diminish rapidly in amplitude as ω becomes large, convergence of this integral is assured from physical considerations.

APPENDIX C

A Comparison of Two Autocorrelation Functions

Two velocity autocorrelation functions were proposed as possible descriptions of the random movements of a subject. They were:

$$\phi_1(\tau) = \sigma^2 e^{-\omega_1 |\tau|} \qquad \phi_2(\tau) = \frac{\sigma^2}{1 + (\omega_1 \tau)^2}$$

Taking the Fourier Transform of these functions gives the following squared velocity spectra (see Appendix B):

$$|V_1(j\omega)|^2 = \frac{2\sigma^2 \omega_1}{\omega_1^2 + \omega^2} \qquad |V_2(j\omega)|^2 = \frac{\pi}{\omega_1} \sigma^2 e^{-\left|\frac{\omega}{\omega_1}\right|}$$

A graph of these two squared velocity spectra as functions of (ω/ω_1) is shown in Fig. 12.

If these squared velocity spectra are substituted into the general power integral and the term $|T(j\omega)|^2$ is assumed to be a ratio of polynomials as for a physical system, then the following integrals result:

$$P_1 = \frac{c}{2\pi} \int_{-\infty}^{\infty} |T(j\omega)|^2 \left[\frac{2\sigma^2 \omega_1}{\omega_1^2 + \omega^2} \right] d\omega$$

$$P_2 = \frac{c}{2\pi} \int_{-\infty}^{\infty} |T(j\omega)|^2 \left[\frac{\pi}{\omega_1} \sigma^2 e^{-\left|\frac{\omega}{\omega_1}\right|} \right] d\omega$$

Inspection of P_1 shows that, for $|T(j\omega)|^2$ a ratio of polynominals, the entire integrand will be a ratio of polynomials and probably can be solved by the method of residues. Inspection of P_2 shows that the exponential term apparently prevents finding a solution in closed form. In order to further evaluate the difficulty and the quantity of the work

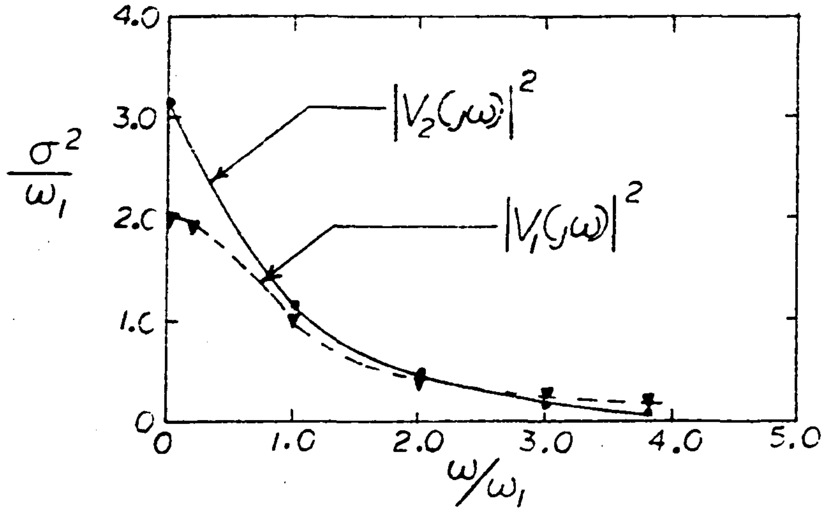

Fig. 12. Two squared velocity spectra

involved in the solution of these integrals, a simplified transfer function,

$$|T(j\omega)|^2 = \left[\frac{\omega^2}{k/m + \omega^2}\right]^2,$$ was substituted into the

integrals and the solutions found for $c^2 = 4km$.

The solution of P_1 is included as a special case of the Type I system solution presented in Appendix D. A graph presenting the results of this evaluation is shown in Fig. 13.

The integral P_2, following substitution of the given transfer function, was rearranged into the following form:

$$P_2 = \sigma^2 \sqrt{km} \int_0^\infty \left[\frac{(\omega/\omega_1)^2}{\frac{k}{\omega_1^2 m} + (\omega/\omega_1)^2}\right]^2 e^{-(\omega/\omega_1)}\, d(\omega/\omega_1)$$

This integral was then referred to the digital computer group for solution using a method of approximation suggested in an article by Ser. (22) The approximation used was:

$$P_2 \doteq \sigma^2 \sqrt{km} \left[1 + \frac{3q}{2}\left\{ \sum_{i=0}^{10} \left(\frac{1}{3}\beta_i - \alpha_i\right)(-1)^i (q-1)^i \right\}\right]$$

$$\text{where } \alpha_n = \int_0^\infty \frac{x\, e^{-x}}{(x+1)^n}\, dx \qquad \beta_n = \int_0^\infty \frac{e^{-x}}{(x+1)^n}\, dx$$

$$\text{and } x = (\omega/\omega_1) \qquad q = \frac{k}{\omega_1^2 m} = (\omega_0/\omega_1)^2$$

A graph presenting the results of this solution for values of q from 0.45 to 1.0 is shown in Fig. 14.

A comparison of the results as presented in Fig. 13 and Fig. 14 shows

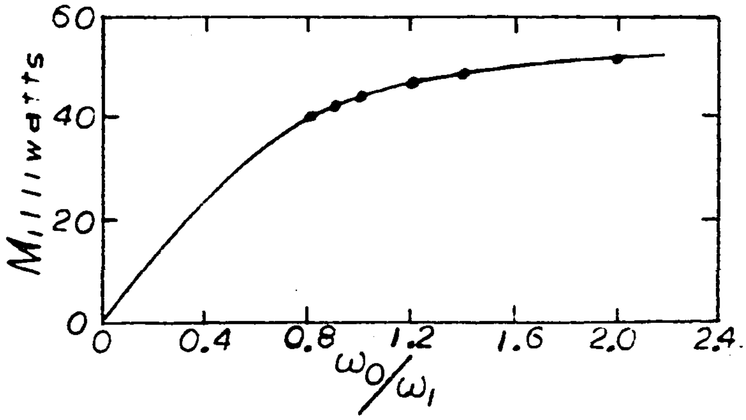

Fig. 13. P_1 versus $(\frac{\omega_0}{\omega_1})$, critical damping

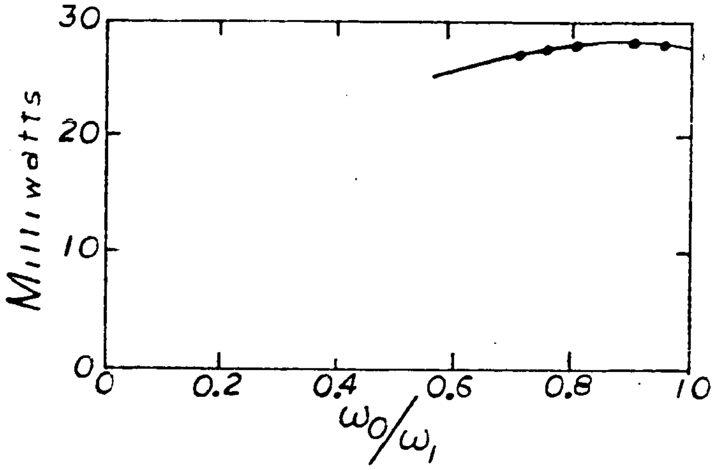

Fig. 14. P_2 versus $(\frac{\omega_0}{\omega_1})$

that the two autocorrelation functions yield values which are of the same order of magnitude and not too significantly different. Because the use of the exponential autocorrelation function led to a much less involved solution process, it was taken as the describing function for the random motion.

APPENDIX D

Evaluation of the Power Integrals P_1 and P_2

The integrals for the power available from both the Type I system and the Type II system are ameanable to solution by the method of residues. These solutions have been tabulated for the general integral of the form

$$I_n = \frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{g_n(x)}{h_n(x)\, h_n(-x)}\, dx$$

where

$$g_n(x) = b_0 x^{2n-2} + b_1 x^{2n-4} + \text{----} + b_{n-1}$$

$$h_n(x) = a_0 x^n + a_1 x^{n-1} + \text{----} + a_n$$

and the roots of $h_n(x)$ all lie in the upper half-plane.

The Type I system power integral is

$$P_1 = \frac{c}{2\pi} \int_{-\infty}^{\infty} \frac{2\sigma^2\, \omega^4\, \omega_1\, d\omega}{[(\frac{k}{m} - \omega^2)^2 + (\frac{\omega c}{m})^2][\omega_1{}^2 + \omega^2]}$$

It can be put in the general form of I_n as follows:

$$P_1 = j2\sigma^2\omega_1 c\left(\frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{\omega^4\, d\omega}{[(\frac{k}{m} - \omega^2) + j\omega(\frac{c}{m})][\omega_1 + j\omega][(\frac{k}{m} - \omega^2) - j\omega(\frac{c}{m})][\omega_1 - j\omega]}\right)$$

$$= j2\sigma^2\omega_1 c\left(\frac{1}{2\pi j}\int_{-\infty}^{\infty} \frac{1}{\omega^3(-j) + \omega^2(-\omega_1 - \frac{c}{m}) + \omega(j\omega_1(\frac{c}{m}) + j\frac{k}{m}) + \omega_1\frac{k}{m}} \cdot \right.$$

$$\left. \frac{1}{\omega^3(j) + \omega^2(-\omega_1 - \frac{c}{m}) + \omega(-j\omega\frac{c}{m} - j\frac{k}{m}) + \omega_1\frac{k}{m}}\quad \omega^4\, d\omega\right)$$

From this expression, the b_n and a_n are

$$g_n(\omega) = b_0\omega^4 + b_1\omega^2 + b_2 \qquad \begin{aligned} b_0 &= 1 \\ b_1 &= 0 \\ b_2 &= 0 \end{aligned}$$

$$h_n(\omega) = a_0\omega^3 + a_1\omega^2 + a_2\omega + a_3 \qquad \begin{aligned} a_0 &= -j \\ a_1 &= -(\omega_1 + \frac{c}{m}) \\ a_2 &= j(\omega_1 \frac{c}{m} + \frac{k}{m}) \\ a_3 &= \omega_1 \frac{k}{m} \end{aligned}$$

The solution of the bracketed function is:

$$I_3 = \frac{-a_2b_0 + a_0b_1 - \frac{a_0a_1b_2}{a_3}}{2\, a_0(a_0a_3 - a_1a_2)} = \frac{-a_2b_0}{2a_0(a_0a_3 - a_1a_2)}$$

$$= \frac{\omega_1 \frac{c}{m} + \frac{k}{m}}{(j2)[\omega_1^2(\frac{c}{m}) + \omega_1(\frac{c}{m})^2 + \frac{ck}{m^2}]}$$

Therefore

$$P_1 = j2\sigma^2\omega_1 cI_3 = j2\sigma^2\omega_1 c \left[\frac{\omega_1 \frac{c}{m} + \frac{k}{m}}{(j2)(\frac{c}{m})(\omega_1^2 + \omega_1(\frac{c}{m}) + \frac{k}{m})}\right]$$

$$= \sigma^2 m\omega_1 \;\frac{\omega_1(\frac{c}{m}) + \frac{k}{m}}{\omega_1^2 + \omega_1(\frac{c}{m}) + \frac{k}{m}} \;.$$

Let $\frac{k}{m} = \omega_0^2$, then

$$P_1 = \sigma^2 m\omega_1 \frac{\omega_1(\frac{c}{m}) + \omega_0^2}{\omega_1^2 + \omega_1(\frac{c}{m}) + \omega_0^2} \;.$$

If, additionally, critical damping is assumed, then $c^2 = 4km$ and, dividing numerator and denominator by ω_1^2

$$P_1 = \sigma^2 m\omega_1 \frac{2(\frac{\omega_0}{\omega_1}) + (\frac{\omega_0}{\omega_1})^2}{1 + 2(\frac{\omega_0}{\omega_1}) + (\frac{\omega_0}{\omega_1})^2} .$$

The Type II system power integral is solved in the same manner. In the standard form given previously

$$P_2 = \frac{c}{2\pi} \int_{-\infty}^{\infty} (\frac{k}{J\ell})^2 \left[\frac{\omega^4}{\omega^8 + (\alpha^2-2\beta)\omega^6 + (\beta^2+2\delta-2\alpha\gamma)\omega^4 + (\gamma-2\delta\beta)\omega^2 + \delta^2} \right] \cdot \left[\frac{2\sigma^2\omega_1}{\omega_1^2 + \omega^2} \right] d\omega.$$

The upper half-plane roots of the denominator are found using the same procedure as before. The second bracketed term is easily solved as $(\omega_1+j\omega)(\omega_1-j\omega)$. The first bracketed term is algebraically more tedious to solve. A method which will somewhat reduce the quantity of work makes use of the fact that the final form is known to be, for the upper half-plane roots,

$$\omega^4 - jA\omega^3 - B\omega^2 + jC\omega + D.$$

When this term is multiplied by its conjugate as follows

$$\begin{aligned}
&[\omega^4 - jA\omega^3 - \omega^2 B + jC\omega + D] \quad [\omega^4 + jA\omega^3 - \omega^2 B - jC\omega + D] \\
&= \omega^8 + \omega^6 (A^2 - B - B) + \omega^4 (D + D - AC - AC + B^2) \\
&\quad + \omega^2 (-BD - BD + C^2) + D^2 ,
\end{aligned}$$

direct comparison with the original denominator shows that it becomes

$$(\omega^4 - j\alpha\omega^3 - \beta\omega^2 + j\gamma\omega + \delta)(\omega^4 + j\alpha\omega^3 - \beta\omega^2 - j\gamma\omega + \delta).$$

Therefore, the power integral may be written as follows:

$$P_2 = (j2)(c\omega_1\sigma^2)\left(\frac{k}{J\ell}\right)^2 I_5 \qquad \text{with } I_5 = \frac{1}{2\pi j}\int_{-\infty}^{\infty} \frac{\omega^4 \, d\omega}{F(j\omega)\,F(-j\omega)}$$

where $F(j\omega) = j\omega^5 + (\omega_1+\alpha)\omega^4 - j(\alpha\omega_1+\beta)\omega^3 - (\beta\omega_1+\gamma)\omega^2 + j(\gamma\omega_1+\delta)\omega + \delta\omega_1$

From this equation, the constants a_n and b_n are

$$\begin{array}{ll}
b_0 = 0 & a_0 = j \\
b_1 = 0 & a_1 = (\omega_1 + \alpha) \\
b_2 = 1 & a_2 = -j(\alpha\omega_1 + \beta) \\
b_3 = 0 & a_3 = -(\beta\omega_1 + \gamma) \\
b_4 = 0 & a_4 = j(\gamma\omega_1 + \delta) \\
 & a_5 = \delta\omega_1
\end{array}$$

The solution of the integral I_5 is

$$I_5 = \frac{M_5}{2a_0\,\Delta_5}$$

where $M_5 = b_0\,(-a_0a_4a_5 + a_1a_4^2 + a_2^2a_5 - a_2a_3a_4)$

$$+ a_0b_1\,(-a_2a_5 + a_3a_4) + a_0b_2\,(a_0a_5 - a_1a_4)$$

$$+ a_0b_3\,(-a_0a_3 + a_1a_2) + \frac{a_0\,b_4}{a_5}\,(-a_0a_1a_5 + a_0a_3^2 + a_1^2a_4 - a_1a_2a_3)$$

and $\Delta_5 = a_0^2a_5^2 - 2a_0a_1a_4a_5 - a_0a_2a_3a_5 + a_0a_3^2a_4 + a_1^2a_4^2 + a_1a_2^2a_5$

$- a_1a_2a_3a_4$.

Because only b_2 is different from zero, M_5 simplifies easily to

$$M_5 = a_0b_2\,(a_0a_5 - a_1a_4) = j\,[j\delta\omega_1 - j(\omega_1+\alpha)(\gamma\omega_1+\delta)]$$

$$M_5 = \gamma\omega_1^2 + \alpha\gamma\omega_1 + \alpha\delta.$$

Evaluation of Δ_5 proceeds by steps as follows.

$$a_0^2 = -1$$

$$a_0^2a_5^2 = -\delta^2\omega_1^2$$

$$a_5^2 = \delta^2\omega_1^2$$

$$a_0a_1 = j(\omega_1 + \alpha)$$

$$a_0a_1a_4a_5 = -(\omega_1 + \alpha)(\gamma\omega_1 + \delta)(\delta\omega_1)$$

$$a_4a_5 = j\delta\omega_1(\gamma\omega_1 + \delta)$$

$$= -(\gamma\,\omega_1^2 + \alpha\gamma\,\omega_1 + \omega_1\delta \div \alpha\delta)(\delta\omega_1)$$

$$= -[\delta\gamma\,\omega_1^3 + (\alpha\delta\gamma + \delta^2)\omega_1^2 + \alpha\delta^2\omega_1]$$

$$-2a_0a_1a_4a_5 = 2\,[\delta\gamma\,\omega_1^3 + (\alpha\delta + \delta^2)\omega_1^2 + \alpha\delta^2\omega_1]$$

$$a_0a_2 = j(-j)(\alpha\omega_1 + \beta) = (\alpha\omega_1 + \beta)$$

$$a_0a_2a_3a_5 = -(\alpha\omega_1 + \beta)(\beta\omega_1 + \gamma)(\delta\omega_1)$$

$$a_3a_5 = -\delta\omega_1(\beta\omega_1 + \gamma)$$

$$= -\delta\omega_1 \ [\alpha\beta\omega_1^2 + (\beta^2 + \alpha\gamma)\omega_1 + \beta\gamma]$$

$$-a_0a_2a_3a_5 = \ [\alpha\beta\delta\omega_1^3 + (\delta\beta^2 + \alpha\gamma\delta)\omega_1^2 + \beta\gamma\delta\omega_1]$$

$$a_0a_4 = j(j)(\gamma\omega_1 + \delta) = -(\gamma\omega_1 + \delta)$$

$$a_0a_3^2a_4 = -(\gamma\omega_1 + \delta)(\beta^2\omega_1^2 + 2\beta\gamma\omega_1 + \gamma^2)$$

$$a_3^2 = (\beta\omega_1 + \gamma)^2 = \beta^2\omega_1^2 + 2\beta\gamma\omega_1 + \gamma^2$$

$$a_0a_3^2a_4 = -[\ \gamma\beta^2\omega_1^3 + (\delta\beta^2 + 2\beta\gamma^2)\omega_1^2 + (2\beta\gamma\delta + \gamma^3)\omega_1 + \delta\gamma^2\ ]$$

$$a_1^2 = (\omega_1 + \alpha)^2 = (\omega_1^2 + 2\alpha\omega_1 + \alpha^2)$$

$$a_4^2 = -(\gamma\omega_1 + \delta)^2 = -\ [\gamma^2\omega_1^2 + 2\gamma\delta\omega_1 + \delta^2]$$

$$a_1^2a_4^2 = -(\gamma^2\omega_1^2 + 2\gamma\delta\omega_1 + \delta^2)(\omega_1^2 + 2\alpha\omega_1 + \alpha^2) = -[\ \gamma^2\omega_1^4 + (2\gamma\delta + 2\alpha\gamma^2)\ \omega_1^3$$

$$+ (4\alpha\gamma\delta + \delta^2 + \alpha^2\gamma^2)\omega_1^2 + (2\alpha\delta^2 + 2\alpha^2\gamma\delta)\omega_1 + \delta^2\alpha^2]$$

$$a_1a_5 = \delta\omega_1(\omega_1 + \alpha) = \delta\omega_1^2 + \alpha\delta\omega_1$$

$$a_2^2 = -(\alpha\omega_1 + \beta)^2 = -(\alpha^2\omega_1^2 + 2\alpha\beta\omega_1 + \beta^2)$$

$$a_1a_2^2a_5 = -\ [(\delta\omega_1^2 + \alpha\delta\omega_1)(\alpha^2\omega_1^2 + 2\alpha\beta\omega_1 + \beta^2)]$$

$$= -\ [\alpha^2\delta\omega_1^4 + (2\alpha\beta\delta + \alpha^3\delta)\omega_1^3 + (\beta^2\delta + 2\alpha^2\beta\delta)\omega_1^2 + (\alpha\delta\beta^2)\omega_1]$$

$$a_1a_2 = -j(\omega_1 + \alpha)(\alpha\omega_1 + \beta) = -j[\alpha\omega_1^2 + (\alpha^2 + \beta)\omega_1 + \alpha\beta]$$

$$a_3a_4 = -j(\gamma\omega_1 + \delta)(\beta\omega_1 + \gamma) = -j\ [\gamma\beta\omega_1^2 + (\delta\beta + \gamma^2)\omega_1 + \delta\gamma]$$

$$-a_1a_2a_3a_4 = [\alpha\omega_1^2 + (\alpha^2 + \beta)\omega_1 + \alpha\beta] \quad [\gamma\beta\omega_1^2 + (\delta\beta + \gamma^2)\omega_1 + \delta\gamma]$$

$$= [(\alpha\beta\gamma)\omega_1^4 + (\alpha\delta\beta + \alpha\gamma^2 + \alpha^2\gamma\beta + \beta^2)\omega_1^3$$

$$+ (\alpha\beta^2\gamma + \alpha\delta\gamma + \alpha^2\delta\beta + \alpha^2\gamma^2 + \delta\beta^2 + \beta\gamma^2)\omega_1^2$$

$$+ (\alpha^2\delta\gamma + \beta\delta\gamma + \alpha\delta\beta^2 + \alpha\gamma^2\beta)\omega_1 + \alpha\beta\delta\gamma\ ] .$$

Now collect the terms as coefficients of powers of ω_1. In the process of collecting these terms, many cancel and a much shortened form results.

$$\omega_1^4 \ (\alpha\beta\gamma - \alpha^2\delta - \gamma^2)$$

$$+ \omega_1^3 \ (\alpha^2\gamma\beta - \alpha^3\delta - \alpha\gamma^2)$$

$$+ \omega_1^2 \ (-\beta\gamma^2 + \alpha\beta^2\gamma - \alpha^2\beta\delta)$$

$$+ \omega_1 \ (-\gamma^3 - \alpha^2\gamma\delta + \alpha\gamma^2\beta)$$

$$+ \ (-\delta\gamma^2 - \delta^2\alpha^2 + \alpha\beta\gamma\delta)$$

$$= (\alpha\beta\gamma - \alpha^2\delta - \gamma^2)(\omega_1^4 + \alpha\omega_1^3 + \beta\omega_1^2 + \gamma\omega_1 + \delta).$$

Combining all of these evaluations,

$$P_2 = (j2)(\omega_1\sigma^2 c)(\frac{k}{J\ell})^2 \frac{\gamma\omega_1 + \alpha\gamma\omega_1 + \alpha\delta}{(j2)(\alpha\beta\gamma-\alpha^2\delta-\gamma^2)(\omega_1^4+\alpha\omega_1^3+\beta\omega_1^2+\gamma\omega_1+\delta)} .$$

Returning to the original parameters

$$\alpha\beta\gamma - \alpha^2\delta - \gamma^2 = \gamma(\alpha\beta-\gamma)-\alpha^2\delta = \frac{c}{J}\left(\frac{g}{\ell} + \frac{k}{m\ell^2}\right)\left(\frac{ck}{J^2}\right) - \left(\frac{c}{J}\right)^2\left(\frac{kg}{J\ell}\right)$$

$$= \left(\frac{c}{J}\right)^2\left(\frac{k}{m\ell^2}\right)\left(\frac{k}{J}\right)$$

$$\gamma\omega_1^2 + \alpha\gamma\omega_1 + \alpha\delta = \left(\frac{c}{J}\right)\left(\frac{k}{m\ell^2} + \frac{g}{\ell}\right)\omega_1^2 + \left(\frac{c}{J}\right)\left(\frac{c}{J}\right)\left(\frac{g}{\ell} + \frac{k}{m\ell^2}\right)\omega_1 + \frac{c}{J}\left(\frac{kg}{J\ell}\right)$$

Substitution of these terms into the power expression results in further cancellation of terms. The final form is then

$$P_2 = \sigma^2 m\omega_1 \frac{\left(\frac{g}{\ell} + \frac{k}{m\ell^2}\right)\omega_1^2 + \frac{c}{J}\left(\frac{g}{\ell} + \frac{k}{m\ell^2}\right)\omega_1 + \frac{kg}{J\ell}}{\omega_1^4 + \left(\frac{c}{J}\right)\omega_1^3 + \left(\frac{k}{J} + \frac{g}{\ell} + \frac{k}{m\ell^2}\right)\omega_1^2 + \frac{c}{J}\left(\frac{g}{\ell} + \frac{k}{m\ell^2}\right)\omega_1 + \frac{kg}{j\ell}} \; .$$

APPENDIX E

An Analog Simulation of the Power Integral

An attempt to investigate the Type II system was made using analog computer techniques. The results obtained by this method were generally inconclusive, primarily because of excessive d-c drift in the operational amplifiers used. Although the results were of questionable value, the complete simulation is described because some of the techniques employed were unusual and because they might be used to advantage in the simulation of a similar problem at some future time. A photograph of the laboratory set-up used is shown in Fig. 15.

The programming of the differential equations on the computer followed techniques known for some time. Because of the drift and instability of the amplifiers, the Type II system was programmed in such a manner that a minimum of interaction between the two closed loops of amplifiers took place. This was not the most efficient connection as can be seen from Fig. 16, but it appeared to be the best possible choice.

The generation of the required random function and the subsequent filtering to produce the required frequency spectrum presented difficulties for two reasons: 1) a random noise generator with an output spectrum flat to zero cycles per second was not available, and 2) a filter with the proper response and a high frequency cut-off of 1.0 cycles per second was not available. An attempt to use frequency scaling was unsuccessful because the multiplying constants for the computer variables became too different in magnitude for the range of the computer amplifiers used.

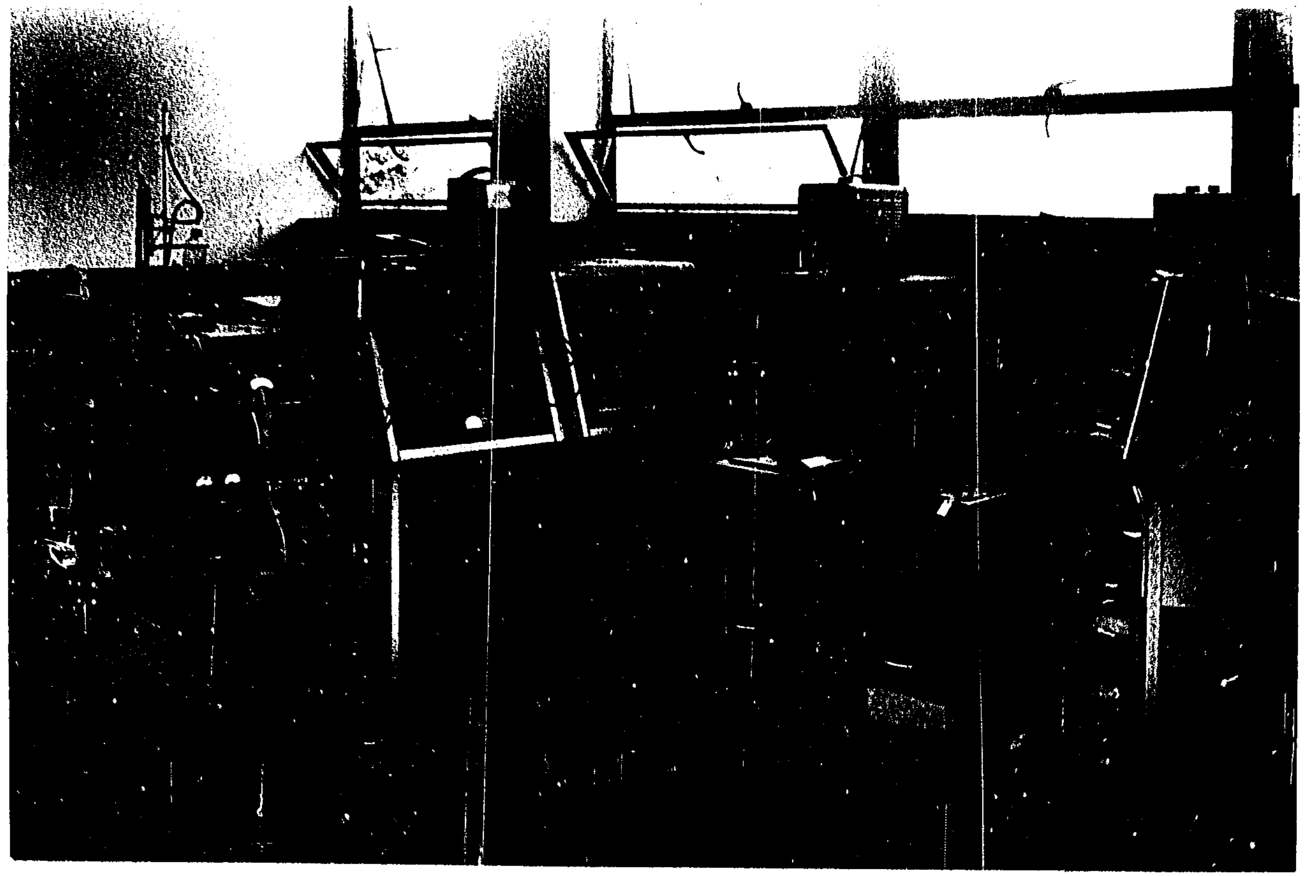

Fig. 15. Analog computer set-up

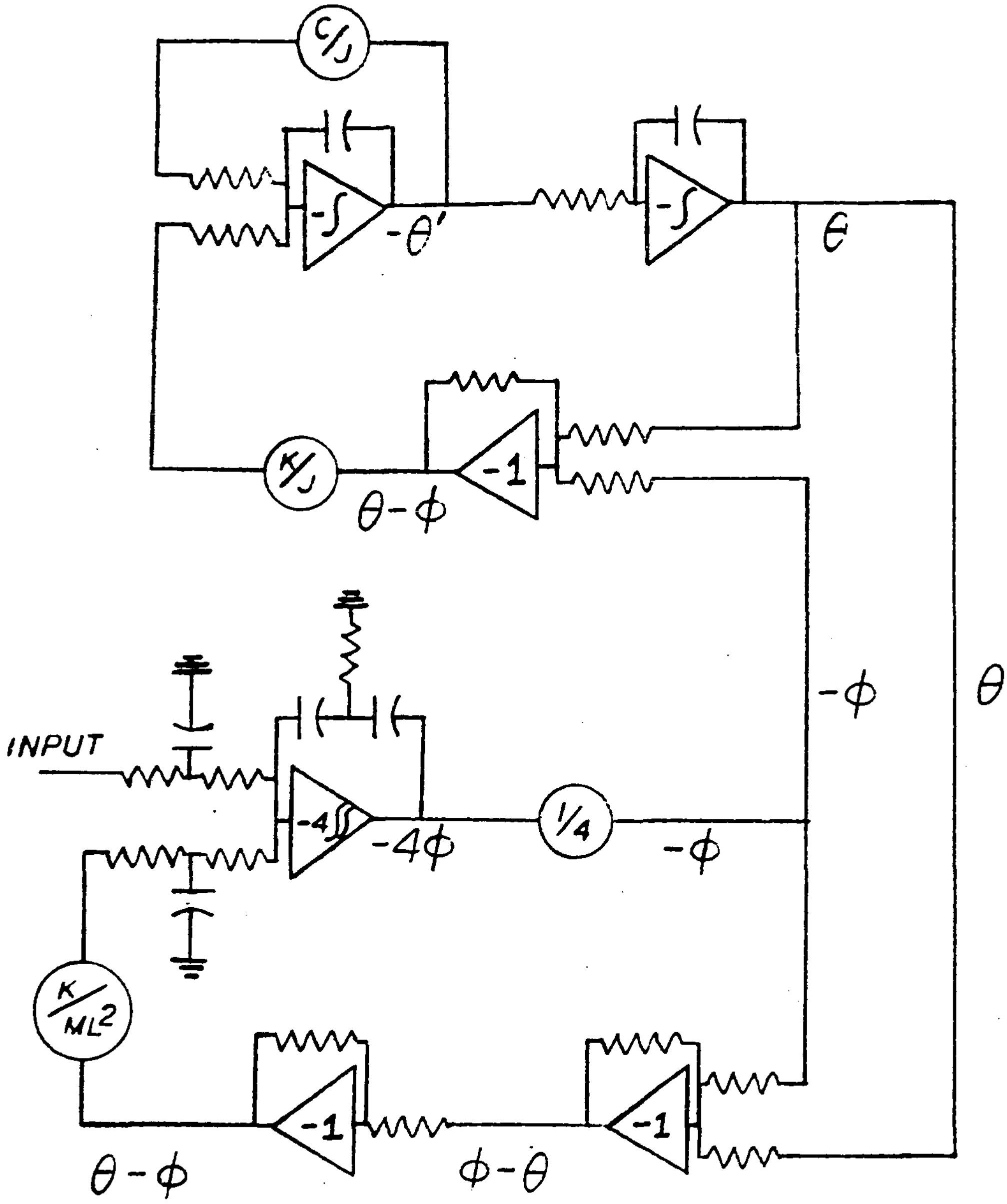

Fig. 16. Analog computer schematic

To overcome these difficulties in obtaining the required driving function for the problem, the output of a noise generator with an output spectrum flat from 30 cycles per second to 20,000 cycles per second was mixed with a constant amplitude, constant frequency 10,000 cycles per second signal from an audio oscillator and the resulting envelope detected with a bridge rectifier. The output of the rectifier was then a signal with a spectrum which was flat from zero cycles per second to nearly 10,000 cycles per second, well above the highest frequency known to be important. The circuitry used is shown in Fig. 17.

The flat spectrum signal was then applied to an active filter to obtain the required spectrum for input to the computer. The required velocity spectrum was found from the squared velocity spectrum,

$$|V(j\omega)|^2 = \frac{2\,\sigma^2\,\omega_1}{\omega_1^2 + \omega^2}$$

This may be written in terms of the complex variables as

$$V(s) = \sigma\sqrt{2\omega_1}\quad\left(\frac{1}{s + \omega_1}\right)$$

From the analog computer diagram, it can be seen that the signal input should be the acceleration spectrum which is

$$X''(s) = s\,V(s) = \sigma\sqrt{2\,\omega_1}\;\left(\frac{s}{s + \omega_1}\right)$$

To obtain this spectrum, an active filter was used. The output of such a filter is given by $X''_0\,(s) = T(s)\cdot X''_{in}(s)$. Since $X''_{in}(s)$ is 1.0 for a flat spectrum, only $T(s)$ is of interest. If an operational amplifier (a high

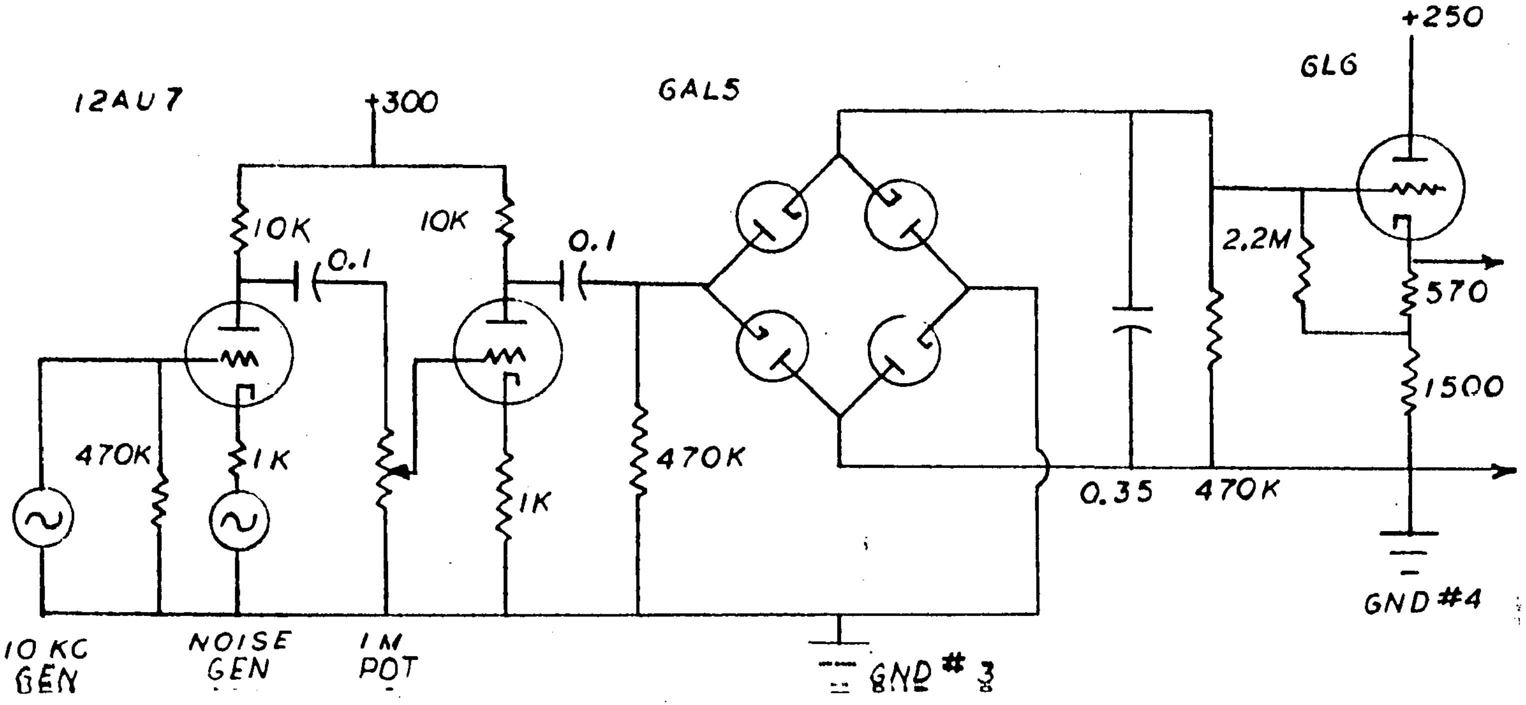

Fig. 17. Driving function circuit

gain d-c amplifier) is used, then

$$T(s) = \frac{Z_2(s)}{Z_1(s)}$$

where $Z_2(s)$ is the feedback impedance from output to input and $Z_1(s)$ is an impedance in series with the applied signal, as shown in Fig. 18a. $T(s)$ is known, is proportional to the acceleration spectrum, and may be written as follows:

$$T(s) = \frac{Z_2(s)}{Z_1(s)} = \frac{s}{s + \omega_1} = \left(\frac{s}{s + a}\right)\left(\frac{s + a}{s + \omega_1}\right)$$

From this, set $Z_1(s) = \frac{s + a}{s} = 1 + a/s$

$$\text{and } Z_2(s) = \frac{s + a}{s + \omega_1} = 1 + \cfrac{1}{\cfrac{s}{a - \omega_1} + \cfrac{\omega_1}{a - \omega_1}}$$

The synthesis was completed by adjusting the impedance level and assuming a value for ω_1 of $\omega_1 = 5$ radians per second. The final filter circuit, with input from the driving function circuit, is shown in Fig. 18b. This circuit gave an output with very nearly the proper spectrum and with a magnitude proportional to the desired output because the factor $\sigma\sqrt{2\omega_1}$ was not included. Because the simulation required calibration of the final power reading, this factor of proportionality was of no significance in this solution.

The method used to obtain a value for the average power dissipation employed operational amplifiers, a diode bridge rectifier, a diode squaring circuit and a heavily damped d-c milliammeter. A voltage proportional to

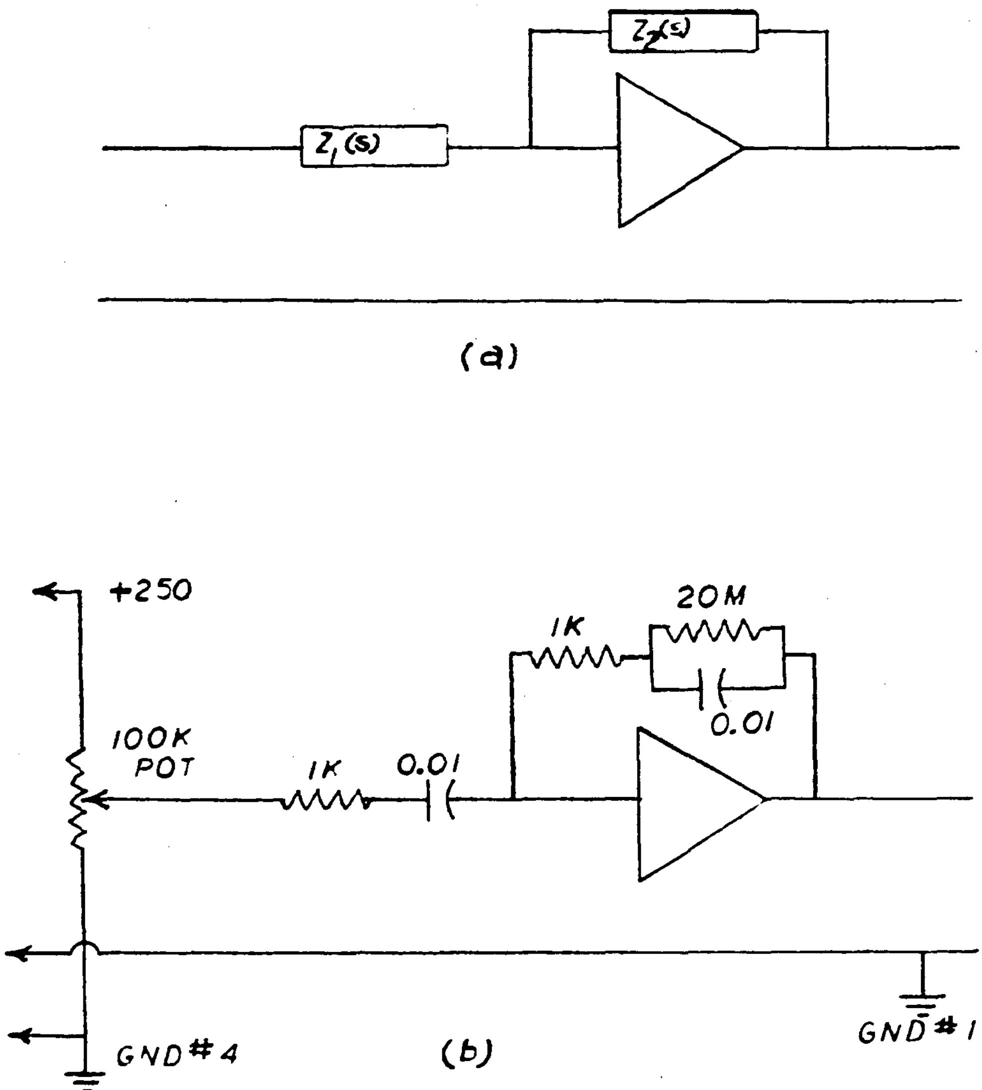

Fig. 18. Filter circuit

$\frac{dy}{dt}$ was available from the computer and was used to obtain a current proportional to the instantaneous power according to the following formula:

$$p(t) = (c \frac{dy}{dt}) \frac{dy}{dt} = (\sqrt{c} \frac{dy}{dt})^2 .$$

The output of the computer, $\frac{dy}{dt}$, was multiplied, by means of a potentiometer, by $\sqrt{c}$ where the appropriate value of c was determined from either the $\frac{c}{J}$ or $\frac{c}{m}$ potentiometer in the computer circuitry. This was possible because the values of J and m were assumed known. To obtain the square of the quantity $\sqrt{c}\ \frac{dy}{dt}$, the voltage proportional to this quantity was first rectified (to obtain the absolute value) and then used as the input to the squaring circuit. Because the voltage levels required in the diode circuits were of the order of volts and those levels available from the computer were of the order of millivolts, amplification of the computer output was obtained by using operational amplifiers. A buffer stage was also necessary because of the different impedance levels between the operational amplifier output and the rectifier input.

The output of the squaring circuit was a current proportional to p(t). This output was obtained using a circuit which performed the following operation: $i_{out} = k(v_{in})^2$ where the constant k was chosen to obtain suitable current levels. This current was then passed through a 5 milliampere d-c ammeter which was heavily damped with 8,000 microfarads of capacitance. The combination of the inertia of the meter movement, the internal resistance of the meter and the large external capacitance gave a very satisfactory intergrating device. As a result the meter deflection was proportional to the average power dissipated in the damping c.

The circuitry used to obtain an amplified voltage proportional to $\sqrt{c}\ \frac{dy}{dt}$ is shown in Fig. 19. In Fig. 20, the buffer stage, the bridge rectifier, the squaring circuit and the averaging circuit are shown. Because balanced circuits were used, the two different power supplies and ground points are indicated in this diagram. In the complete simulation problem, a total of four different power supplies and grounds were necessary, as noted on the diagrams.

In order to obtain quantitative information from this simulation, the meter deflections had to be calibrated. This was accomplished by assuming a problem for which the analytical solution was easily obtained, programming this problem on the computer and then recording the meter reading for a value of power found analytically. A typical calibration used was 1.0 milliamperes equals 14 milliwatts.

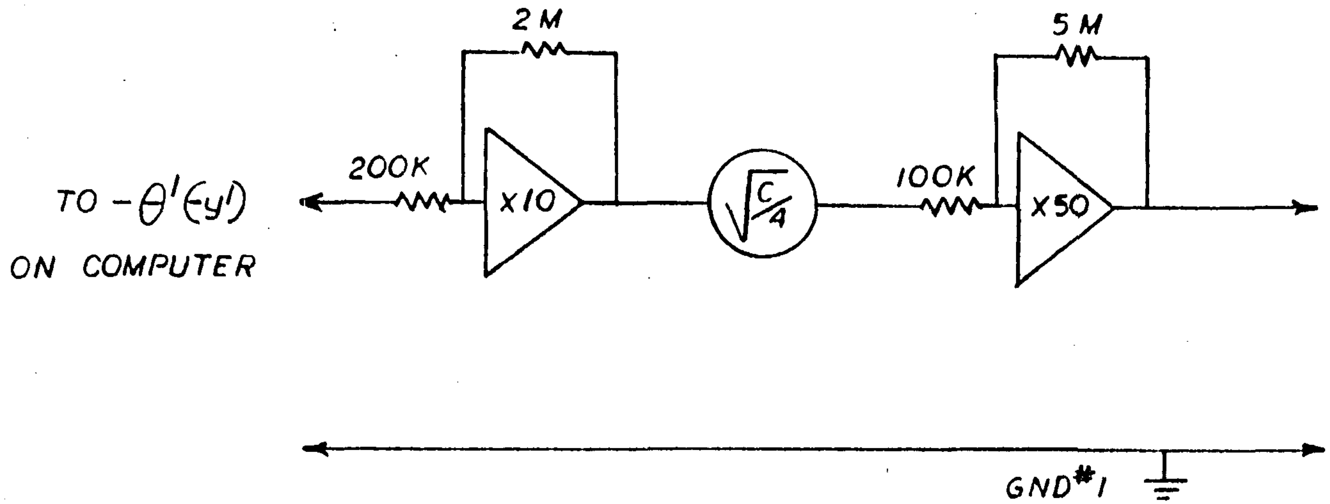

Fig. 19. Circuit for C $\frac{dy}{dt}$ and amplifiers

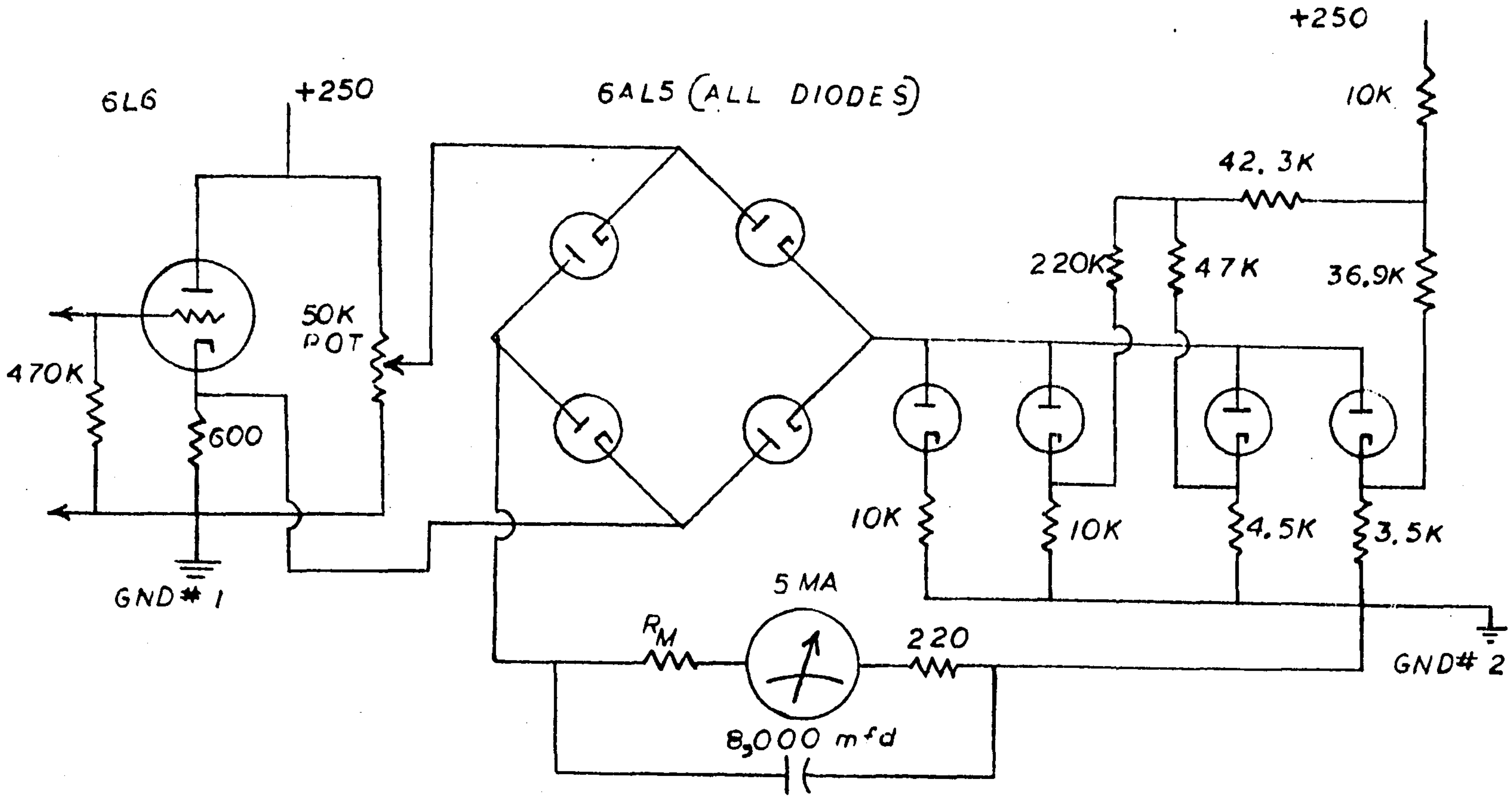

Fig. 20. Squaring and averaging circuit

APPENDIX F

Calculations of Power Output of the Heart

Dukes (5) presents the following equation for the calculation of the work done by the heart:

$$W = 7/6\ QR + mV^2/g \text{ kilogram-meters}$$

The first term on the right represents the work done in moving a given quantity of blood, Q, against the resisting pressure, R, and the second term represents the work done in giving this blood mass, m, a velocity, V. The constant g is the acceleration of gravity, equal to 9.8 meters per second per second. In the metric system, the force unit newton and weight unit kilogram show a one to one correspondence in value, thus, the energy is actually equal to the number of kilogram-meters. Power, that is, energy per unit time, is then given in watts if the units are made newton-meters per second. Since the usual quantitative unit of blood volume flow is liters per minute, and since m equals volume flow times the specific gravity of the blood, the following modified equation will give the power output of the heart in watts if: Q is the minute volume of blood flow of the heart in liters; R is the blood pressure in meters of blood; V is the blood velocity in meters per second; m is the blood mass moved per minute; and g is the gravitational constant. The 1/60 multiplier is the conversion factor from minutes to seconds so that the resulting unit is watts.

$$P = 1/60 \left[7/6\ QR + mV^2/g \right] \text{ watts}$$

For the cow, from data in Dukes (5),

Q = 35 liters per minute

R = 120 mm Hg = (120)(13)/1000 meters of blood

m = Q x sp. gr. = 35 (1.06) kilograms per minute

V = 1.5 meters per second

Substituting these values into the above equation, the power output of the heart of a cow is approximately 12 watts.

For the dog, from data in Dukes (5),

Q = 1.45 liters per minute

R = 160 mm Hg = (160)(13)/1000 meters of blood

m = 1.45 (1.06) kilograms per minute

V = 0.124 meters per second

Substituting these values into the above equation, the power output of the heart of the dog is approximately 0.6 watt.